always true

Boolean Logic

Objects: statements, valued either true or false

true	false
T	F
1	0
Above	Below

Above ↘ Below ↙ a threshold voltage

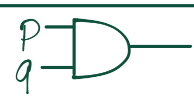
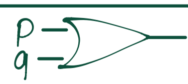
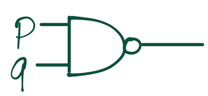
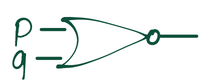
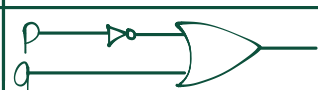
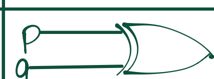
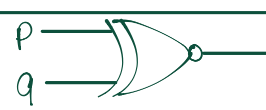
→ thresholds (rounding to 0 or 1)

variables: p, q, r

equivalence relation: Logically equivalent, $\Longleftrightarrow$

Two statements that produce all the same truth values for every possible combinations of inputs

Operation:

Name	mathematical notation	electronics symbol	Rule
Negation/Not	$\sim p$, $\neg p$, p^{-1}, $\overline{p}$		Not T is F Not F is T
Conjunction/And	$p \wedge q$		T and T is T; otherwise F
Disjunction/OR	$p \vee p$		F or F is F; otherwise T
NAND	$\sim(p \wedge q)$		T NAND T is F; otherwise T
NOR	$\sim(p \vee q)$		F nor F is T; otherwise F
Conditional/ If..., then... implies	$p \to q$ $p \to q \Longleftrightarrow \sim p \vee q$		T→F is F, otherwise T *Does not commute
XOR Exclusive OR	$p \oplus q$ $p \otimes q$		same value is F opposite value is T Ex: Both T is F Both F is F T XOR F is T F XOR T is F
Biconditional if and only if (IFF)	$p \longleftrightarrow q \Longleftrightarrow (p \to q) \wedge (q \to p)$		same value is T opposite value is F
XNOR	$\sim(p \otimes q)$		

Truth tables

AND 1000=8

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

NEGATION

p	$\sim p$
T	F
F	T

OR 1110=14

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

NAND 0111=7

p	q	$\sim(p \wedge q)$
T	T	F
T	F	T
F	T	T
F	F	T

NOR 0001=1

p	q	$\sim(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

CONDITIONAL 1011=11

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

XOR 0110=6

p	q	$p \otimes q$
T	T	F
T	F	T
F	T	T
F	F	F

XNOR 1001=9

p	q	$\sim(p \otimes q)$
T	T	T
T	F	F
F	T	F
F	F	T

BICONDITIONAL 1001=9

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$\sim p \vee ((q \to p) \wedge q)$

p	q	$\sim p$	$q \to p$	$(q \to p) \wedge q$	$\sim p((q \to p) \wedge q)$
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	F	I

<u>vocab</u>

conditional: $p \to q$

converse: $q \to p$

inverse: $\sim p \to \sim q$

contrapositive: $\sim q \to \sim p$

tautology: a statement that always evaluates to true ($p \vee \sim p$ is easiest)

transistor: voltage controlled switch - 

Practice

~(pvq)

p	q	pvq	~(pvq)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

pV~p

p	~p	pV~p
T	F	T
F	T	T

tautology

~p∧~q

p	q	~p	~q	~p∧~q
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

q→(pvq)

p	q	pvq	q→(pvq)
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Specific String Solving

and

```
  1010 1101
∧ 0110 1101
  0010 1101
```

XOR

```
  1010 1101   message
⊗ 0110 1101   key
  1100 0000   cypher text
⊗ 0110 1101   key
  1010 1101   message
```

extra

XOR encryption

let p = 1000 1110
q = 0110 0110

a) p→~q

~q = 1001 1001

```
  1000 1110
→ 1001 1001
  1111 1001
```

b) (pvq)v~q

```
  1000 1110
v 0110 0110
  1110 1110
```

```
  1110 1110
v 1001 1001
  1111 1111
```