

Unit 3 Part 1: Boolean Logic and Logic Circuits

STUDY GUIDE

1. Define, write the mathematical notation, and give an example of each of the following.

Logically Equivalent:

Two statements that produce the same truth table



Tautology:

All outputs are true regardless of inputs. A statement that always evaluates the true.

$$p \vee \sim p \quad | \quad \text{Anti-taut.} \quad p \wedge \sim p$$

Inverse:

if $S \rightarrow t$, then $\sim S \rightarrow \sim t$ / negation of if then

Converse:

(0-together) $t \rightarrow S$ / Switch if, then

Contrapositive:

$$\sim t \rightarrow \sim S$$

2. Fill in the following truth table.

p	q	$p \wedge q$	$q \rightarrow p$	$\sim (q \rightarrow p)$	$(p \wedge q) \leftrightarrow \sim (q \rightarrow p)$
T	T	T	T	F	F
T	F	F	T	F	T
F	T	F	F	T	F
F	F	F	T	F	T

3. Construct a truth table for $(p \leftrightarrow q) \vee (p \rightarrow (\sim q))$

p	q	$\sim q$	$p \rightarrow \sim q$	$p \leftrightarrow q$	$(p \leftrightarrow q) \vee (p \rightarrow (\sim q))$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	T	T	T

(tautology)

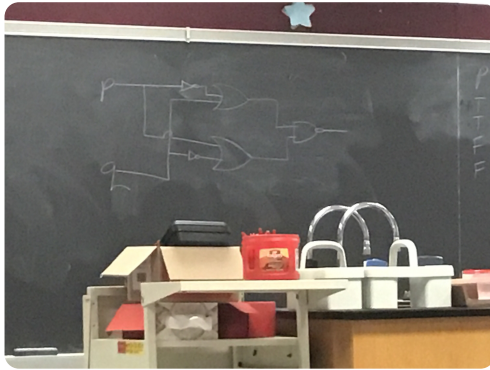
4. State the eight logical equivalences presented in class.

$$\begin{aligned}
 P \rightarrow Q &\Leftrightarrow \sim P \vee Q \\
 P \leftrightarrow Q &\Leftrightarrow \sim(P \otimes Q) \\
 \neg(P \wedge P) &\Leftrightarrow \sim P \\
 \neg(P \vee Q) &\Leftrightarrow \sim P \wedge \sim Q \\
 \neg(P \wedge Q) &\Leftrightarrow \sim P \vee \sim Q \\
 P \rightarrow Q &\Leftrightarrow \sim Q \rightarrow \sim P \\
 \neg(\neg P) &\Leftrightarrow P \\
 P \rightarrow \sim Q &\Leftrightarrow Q \rightarrow P \\
 \end{aligned}$$

(Conditional)
(Contrapositive)
Inverse
(Converse)

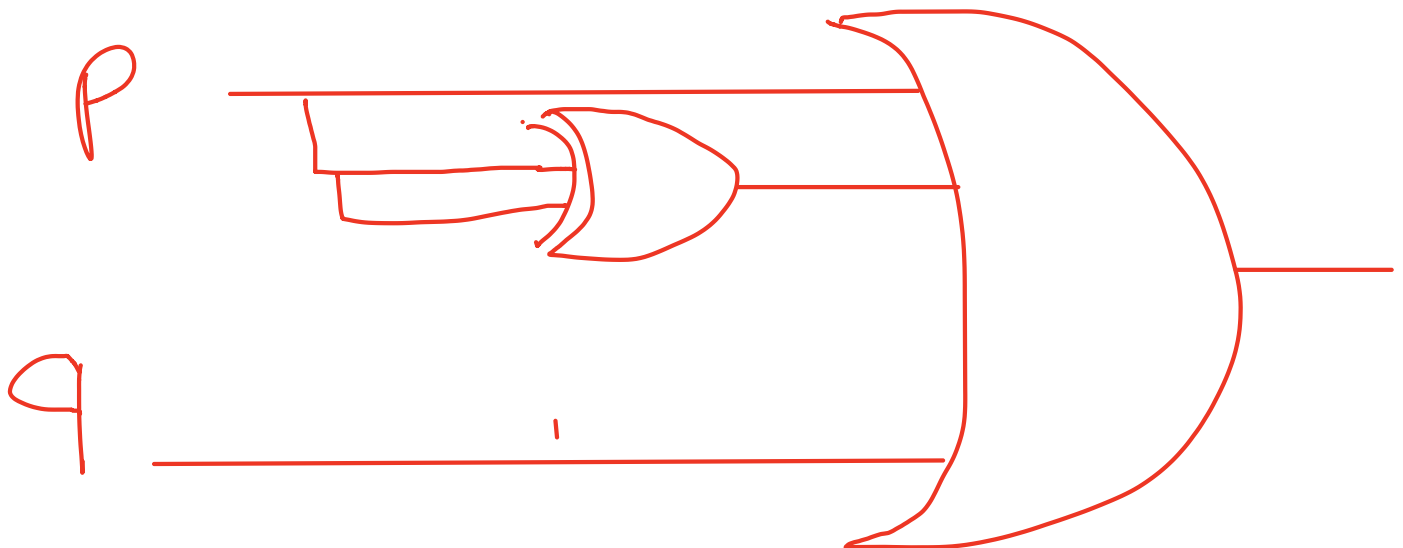
5. Draw a circuit diagram for $\sim((\sim p \vee q) \wedge (p \vee \sim q))$.

Start at end

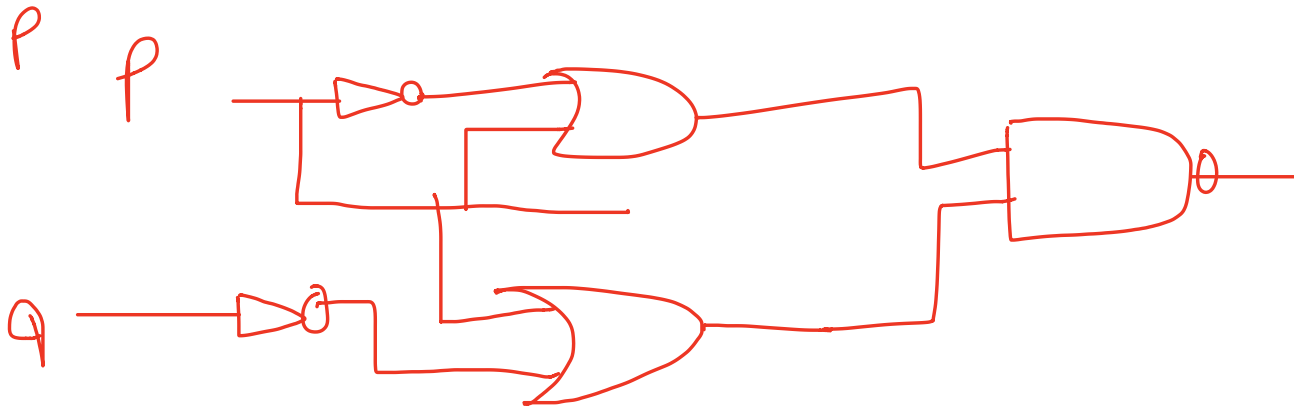
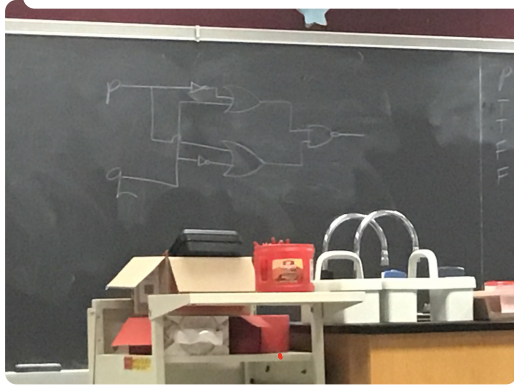


6. Rewrite the statement to eliminate the use of the biconditional and then draw a circuit diagram for $(p \vee q) \leftrightarrow \sim p$.

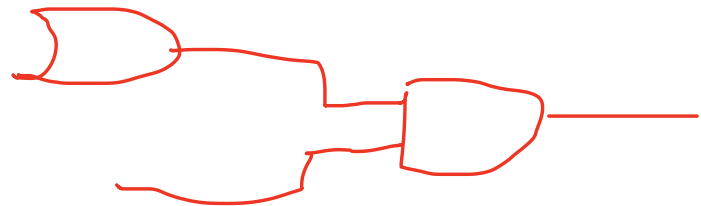
$$\neg((p \vee q) \otimes \sim p)$$



5. $\sim((\sim p \vee q) \wedge (p \vee \sim q)).$



$\sim((\sim p \vee q) \wedge (p \vee \sim q)).$



$$S \rightarrow t \Leftrightarrow \sim S \vee t$$

$$\downarrow \quad \downarrow$$

$$\sim q \quad \sim p$$

$$\sim q \rightarrow \sim p$$

7. Simplify $\sim(\sim q \rightarrow (\sim p))$ using logical equivalences and then draw a circuit diagram of the simplified statement.

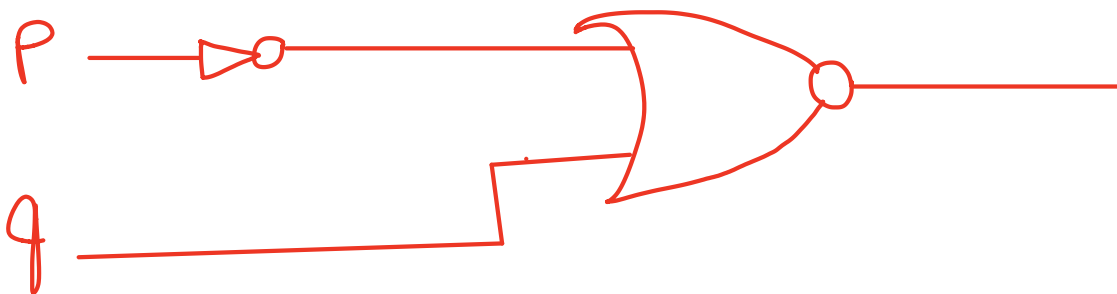
Simplified statement:

$$P \rightarrow q \Leftrightarrow \sim p \vee q$$

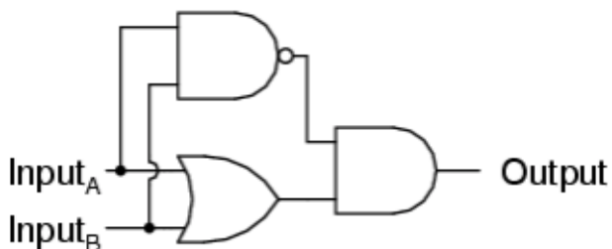
$$\sim(q \vee \sim p)$$

Circuit Diagram:

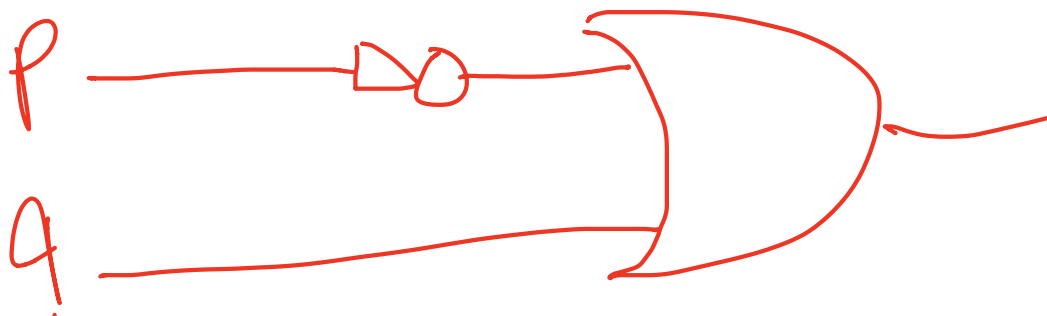
F	F	T	F
T	F	F	T
F	T	T	F
T	T	T	f



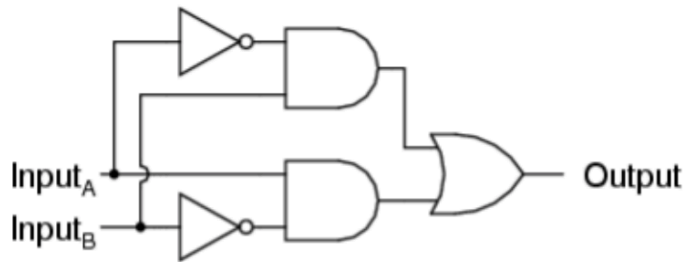
8. Write a logic statement for the following circuit diagram.



$$(A \leftrightarrow B) \wedge (A \vee B)$$



9. Use this circuit diagram to answer the following questions.



a. Write the logic statement the circuit represents.

$$(\neg a \wedge b) \wedge (\neg b \wedge a)$$

b. Create a truth table for the statement.

a	b	$\neg a$	$\neg b$	$\neg a \wedge b$	$\neg b \wedge a$	$(\neg a \wedge b) \wedge (\neg b \wedge a)$
t	t	f	f	f	f	f
t	f	f	t	f	t	f
f	t	t	f	t	f	f
f	f	t	t	f	f	f

c. Find a simpler statement that is logically equivalent to the statement in part a.

$$\neg(p \vee q)$$

d. Draw a circuit diagram of the statement in part c.

