

Number bases

Decimal: Base 10

Binary: Base 2

Hexadecimal: Base 16

Converting from
Base "B" expansion

1. Write the power
of "B" expansion

2. Perform the decimal
arithmetic

Power of 10 expansion is
exponents

Ex: $\begin{array}{ccccccc} 2 & 3 & 7 & 1 & 0 \\ \hline 10^4 & 10^3 & 10^2 & 10^1 & 10^0 \end{array}$

$$2 \times 10^4 + 3 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 0 \times 10^0$$

Ex:
(other
bases)

20341_{five} or (20341)₅
how to tell
what base

$$\begin{array}{ccccc} 2 & 0 & 3 & 4 & 1 \\ \hline 5^4 & 5^3 & 5^2 & 5^1 & 5^0 \end{array}$$

$$2 \times 5^4 + 0 \times 5^3 + 3 \times 5^2 + 4 \times 5^1 + 1 \times 5^0$$

$$1250 + 0 + 75 + 20 + 1$$

$$= 1346$$

Practice

1. $\begin{array}{ccccccc} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ \hline 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 & 2^{-1} \end{array}$

$$1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$= 128 + 0 + 32 + 16 + 0 + 4 + 2 + 1$$

$$= 183$$

2. $\begin{array}{ccc} 6 & 7 & 0 \\ \hline 8^2 & 8^1 & 8^0 \end{array}$

$$6 \times 8^2 + 7 \times 8^1 + 0 \times 8^0$$

$$= 384 + 56 + 0$$

$$= 440$$

3. $\begin{array}{cccc} A & 1 & 0 & C \\ \hline 16^3 & 16^2 & 16^1 & 16^0 \end{array}$

$$10 \times 16^3 + 1 \times 16^2 + 0 \times 16^1 + 12 \times 16^0$$

$$= 40,960 + 256 + 0 + 12$$

$$= 41,228$$

4. $\begin{array}{ccc} 1 & 2 & 3 \\ \hline 4^2 & 4^1 & 4^0 \end{array}$

$$1 \times 4^2 + 2 \times 4^1 + 3 \times 4^0$$

$$= 16 + 8 + 3$$

$$= 27$$

Base 16 vs. Base 10

base 10	base sixteen		base 10	base sixteen
0	0		8	8
1	1		9	9
2	2		10	A
3	3		11	B
4	4		12	C
5	5		13	D
6	6		14	E
7	7		15	F
			16	10

5. $\begin{array}{ccc} 2 & 1 & 3 \\ \hline 16^2 & 16^1 & 16^0 \end{array}$

$$2 \times 16^2 + 1 \times 16^1 + 3 \times 16^0$$

$$= 512 + 16 + 3$$

$$= 541$$

6. $\begin{array}{cccc} 5 & 2 & 0 & 1 \\ \hline 6^3 & 6^2 & 6^1 & 6^0 \end{array}$

$$5 \times 6^3 + 2 \times 6^2 + 0 \times 6^1 + 1 \times 6^0$$

$$= 1,080 + 72 + 0 + 1$$

$$= 1,153$$

7. $\begin{array}{ccccccc} 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ \hline 2^4 & 2^3 & 2^2 & 2^1 & 2^0 & 2^{-1} & 2^{-2} \end{array}$

$$2^4 + 2^3 + 0 + 2^1 + 2^0 + 2^{-1} + 0 + 2^{-2} + 2^{-4}$$

$$16 + 8 + 0 + 2 + 1 + \frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16} = 27 \frac{11}{16}$$

$$= 27.68$$

Converting out of base 10 (decimal)

into base "B"

1. Write the powers of b till you reach a value larger than the given number

→ 2941_{ten} to Base 8

$$\begin{aligned} 8^0 &= 1 \\ 8^1 &= 8 \\ 8^2 &= 64 \\ 8^3 &= 512 \\ 8^4 &= 4096 \end{aligned}$$

2. Create the same number of place value slots as the exponent on step 1

→ _ _ _ _

3. Divide the given number by the largest power of b smaller than the number

→ The largest power of 8 smaller than 2941 is $8^3 = 512$

$$\frac{2941}{512} = 5 \text{ R } 381$$

4. Write the quotient in the left most open spot

5. Repeat steps 3+4 using the remainder as the "given number" until the remainder is zero

$$\underline{5} \quad \underline{5} \quad \underline{7} \quad \underline{5}$$

$$\frac{381}{64} = 5 \text{ R } 61$$

$$\begin{array}{r} 64 \overline{) 381} \\ \underline{320} \\ \text{R } 61 \end{array}$$

$$\frac{61}{8} = 7 \text{ R } 5$$

6. Fill zeros if needed

$$\frac{5}{1} = 5 \text{ R } 0$$

2941_{ten} = 5,575_{eight}

Practice

1368_{ten} to binary

$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 4$$

↓

$$2^{10} = 1024$$

$$2^{11} = 2048$$

1 0 _ _ _ _ _

$$\begin{array}{r} 1024 \overline{) 1368} \\ \underline{1024} \\ \text{R } 344 \end{array}$$

$$1. \begin{array}{cccc} 2 & 1 & 3 & 4 \\ \hline 6^3 & 6^2 & 6^1 & 6^0 \end{array} \text{six}$$

$$2 \times 6^3 + 1 \times 6^2 + 3 \times 6^1 + 4 \times 6^0$$

$$= 432 + 36 + 18 + 4$$

$$= 490_{\text{ten}}$$

2. 678_{ten} to Base 6

$$\begin{aligned} 6^0 &= 1 \\ 6^1 &= 6 \\ 6^2 &= 36 \\ 6^3 &= 216 \\ 6^4 &= 1,296 \end{aligned}$$

$$\begin{array}{cccc} 3 & 0 & 5 & 0 \end{array}$$

$$\begin{array}{r} 216 \overline{) 678} \\ \underline{648} \\ R30 \end{array}$$

$$\begin{array}{r} 36 \overline{) 30} \\ \underline{R30} \end{array}$$

$$\begin{array}{r} 6 \overline{) 30} \\ \underline{R0} \end{array}$$

$\downarrow$ 3,050_{six}

3. $3A0_{\text{six}}$
to base 10

$$\begin{array}{ccc} 3 & A & 0 \\ \hline 6^2 & 6^1 & 6^0 \end{array}$$

$$3 \times 6^2 + 10 \times 6^1 + 0 \times 6^0$$

$$= 768 + 160 + 0$$

$$= 928_{\text{ten}}$$

$$\begin{array}{ccccccc} & 3 & A & 0 & & & \\ & 0011 & 1010 & 0000 & & & \end{array}$$

to binary

$$928_{\text{ten}}$$

$$\begin{array}{l} 2^0 = 1 \\ 2^1 = 2 \\ 2^2 = 4 \\ 2^3 = 8 \\ 2^4 = 16 \\ 2^5 = 32 \\ 2^6 = 64 \\ 2^7 = 128 \\ 2^8 = 256 \\ 2^9 = 512 \\ 2^{10} = 1,024 \end{array}$$

$$\begin{array}{r} \begin{array}{ccccccccccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \\ 512 \overline{) 928} \\ \underline{R416} \end{array}$$

$$\begin{array}{r} 256 \overline{) 416} \\ \underline{R160} \end{array}$$

$$\begin{array}{r} 128 \overline{) 160} \\ \underline{R32} \end{array}$$

$$\begin{array}{r} 64 \overline{) 32} \\ \underline{R32} \end{array}$$

$$\begin{array}{r} 32 \overline{) 32} \\ \underline{R0} \end{array}$$

$$\begin{array}{r} 16 \overline{) 0} \\ \underline{R0} \end{array}$$

$\downarrow$ 1110 1000 00_{two}

$$4. \begin{array}{ccccccc} & 8 & & 7 & & & 2 \\ 10 & 11 & 0111 & 0010 & & & \\ \hline 1 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{array}$$

to base 10

$$2^{11} + 2^9 + 2^8 + 2^6 + 2^5 + 2^4 + 2^1$$

$$4,096 + 1,024 + 512 + 128 + 64 + 32$$

$$= 5,860_{\text{ten}}$$

2,930 (i started wrong)

to base 16

$$2930$$

$$\begin{array}{l} 16^0 = 1 \\ 16^1 = 16 \\ 16^2 = 256 \\ 16^3 = 4,096 \\ 16^4 = 65,536 \end{array}$$

$$\begin{array}{r} \begin{array}{ccc} 8 & 7 & 2 \\ \hline 256 \overline{) 2930} \\ \underline{R114} \end{array} \\ 16 \overline{) 114} \\ \underline{R7} \\ 16 \overline{) 7} \\ \underline{R7} \\ 16 \overline{) 0} \\ \underline{R0} \end{array}$$

872_{sixteen}

Hex to binary

1. $2ADF$

$$0010 \ 1010 \ 0000 \ 1111$$

2. 10

$$10000$$

3. $6D4$

$$0110 \ 1101 \ 0100$$