Solving Logic Problems using Truth Tables

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Defining a Logic Problem

Selecting and Using an appropriate Truth Table

Using Truth Tables to determine the solution to a Logic Problem

Implementing the solution using hardware (engineering)

Implementing the solution using software (computer science)

Defining a Logic Problem

The assumption for this lesson is that a logic problem consists of

- Binary inputs – all inputs are yes/no, on/off, or true/false inputs
- Binary outputs – the output(s) are yes/no, on/off, or true/false
- The state of the output is based on the state of the inputs

Example: If the burglar alarm is engaged and a window is open or motion is detected, sound the alarm.

Identify the Inputs and Outputs

For each input and output, identify what constitutes on and off (true and false).

Example: If the burglar alarm is turn on and a window is open or motion is detected, sound an audible alarm.

Inputs

- Burglar Alarm: 1 – Alarm On, 0 – Alarm Off
- Window: 1 – Closed, 0 – Open
- Motion Detector: 1 – Motion, 0 – No Motion

Outputs

- Alarm: 1 – On, 0 – Off

Select an appropriate Truth Table

The appropriate Truth Table is based on the number of inputs

Two Inputs

X	Y	Out
0	0	
0	1	
1	0	
1	1	

Three Inputs

X	Y	Z	Out
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Four Inputs

W	X	Y	Z	Out
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

Fill in the Truth Table

X	Y	Z	Out
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

X = Burglar Alarm: 1 – Alarm On, 0 – Alarm Off
Y = Window: 1 – Closed, 0 – Open
Z = Motion Detector: 1 – Motion, 0 – No Motion

Out = Alarm: 1 – On, 0 - Off

Document the Solution

X	Y	Z	Out
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Identify the inputs that cause the output to be 1. These are the min-terms.

Write the equation for each min-term.

If an input is 1, write the variable name. If the input is 0, write the inverse of the variable name.

If X = 0, use $\bar{X}$

If X = 1, use X

Inputs are AND'ed (the first min-term is $X \ * \ \bar{Y} * \bar{Z}$)

Combine min-terms to form the equation

Min-terms are OR'ed

The full solution is $X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ$

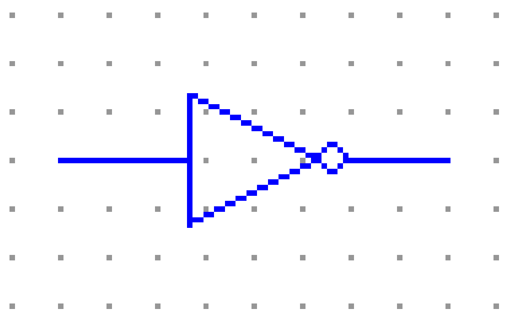
Simplify the Solution

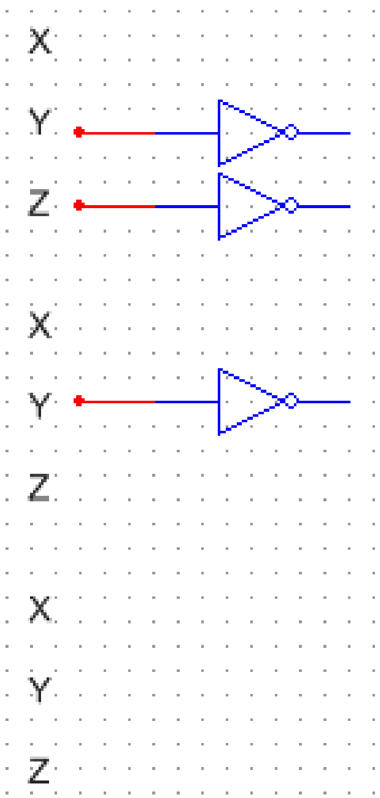
Solutions can be simplified using Boolean Algebra or Karnaugh Mapping. These topics are beyond the scope this lesson, therefore this lesson will use unsimplified logic expressions.

Test the Solution (Hardware)

$$X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ$$

1) Invert any inputs that have to be inverted using an Inverter Gate

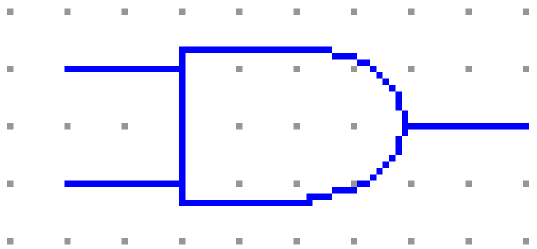


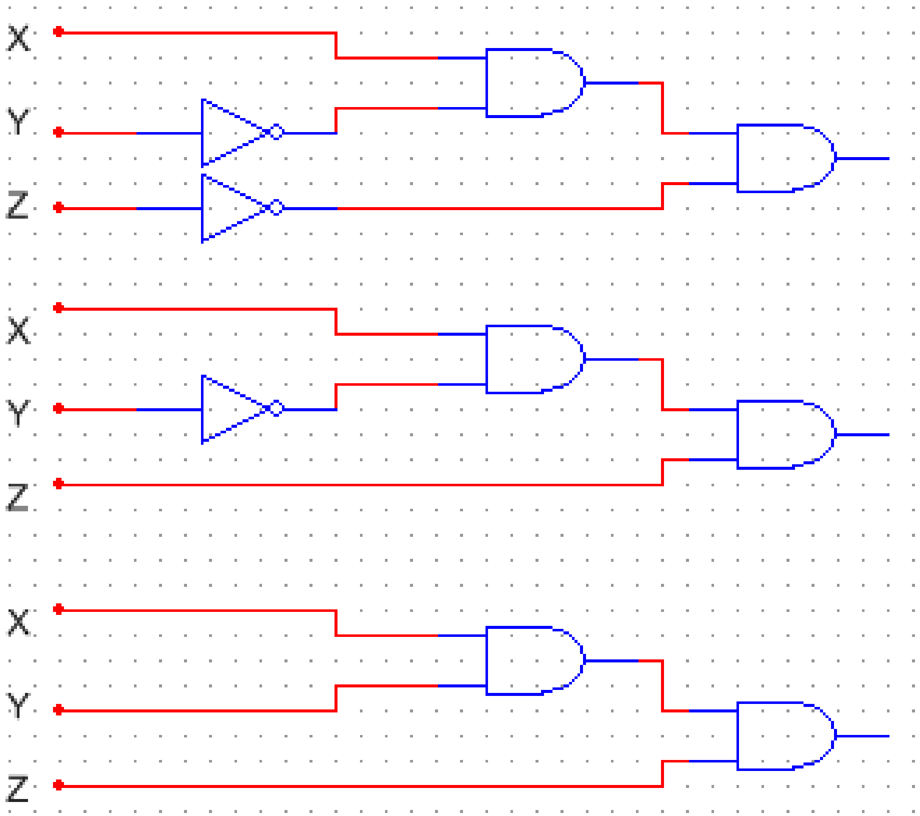


Test the Solution (Hardware)

$$X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ$$

2) AND the inputs to form a min-term using an AND Gate

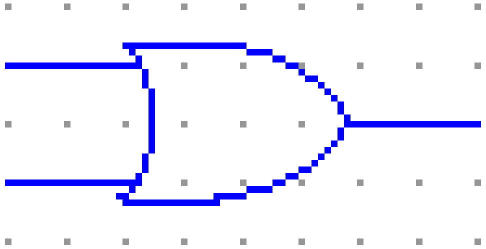


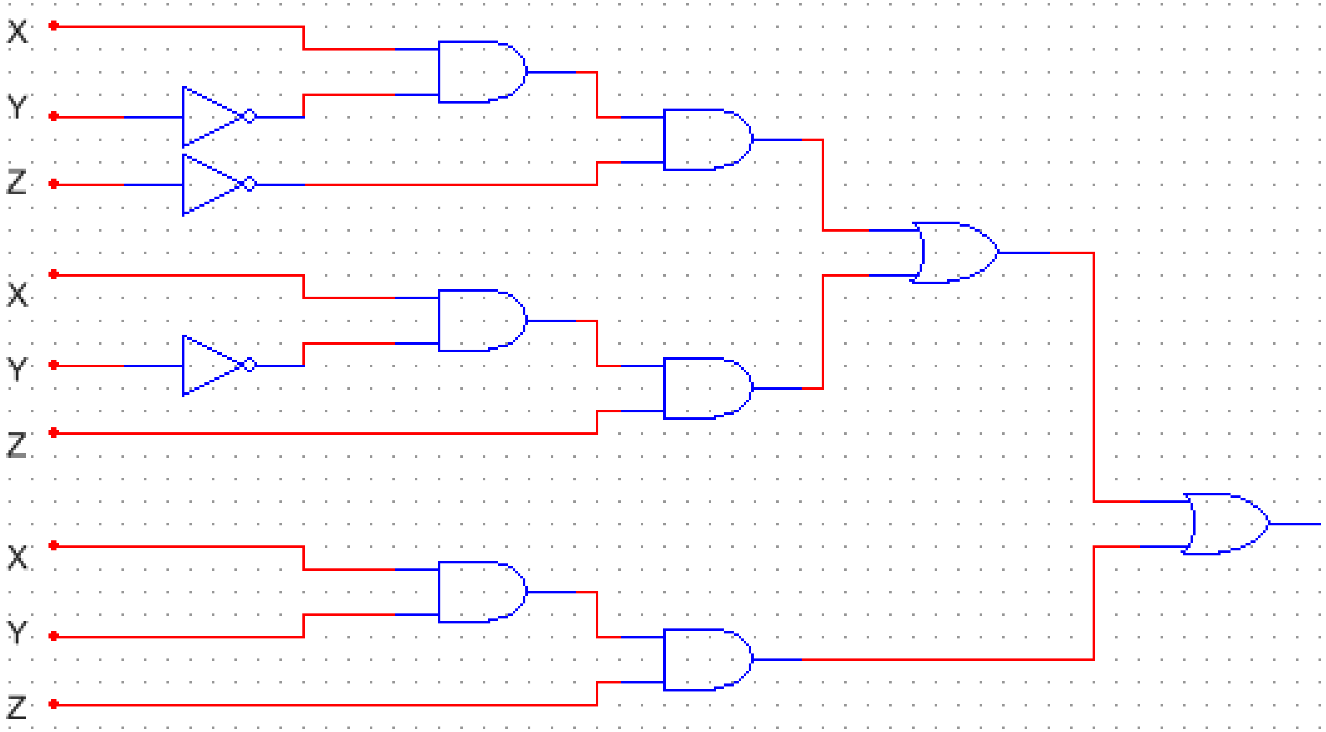


Test the Solution (Hardware)

$$X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ$$

3) OR the min-terms to form the solution using an OR Gate

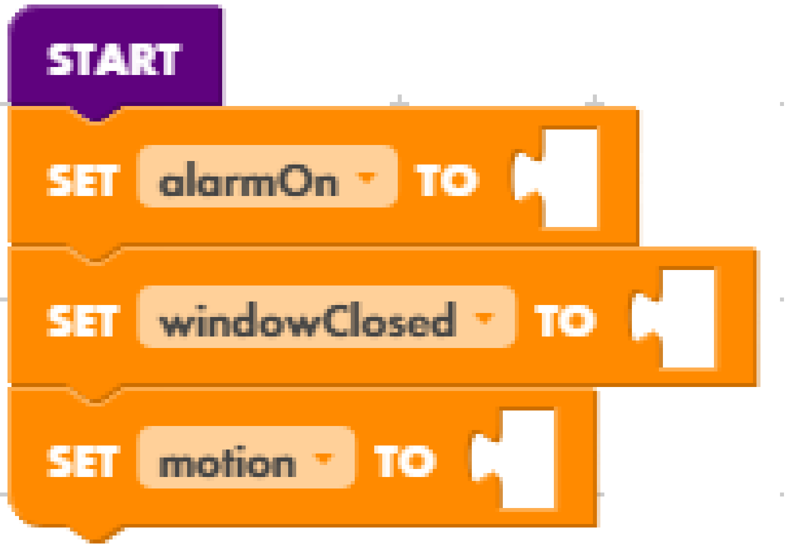


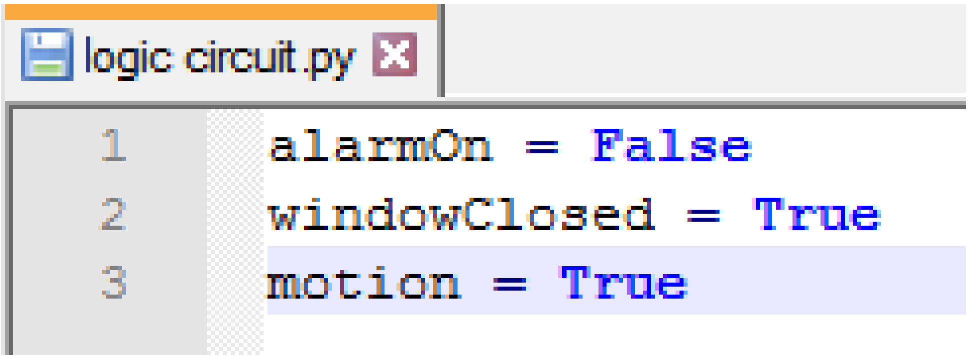


Test the Solution (Software)

$$X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ$$

o) Define variables for each input.
A common practice is name the variable after the positive (true) state

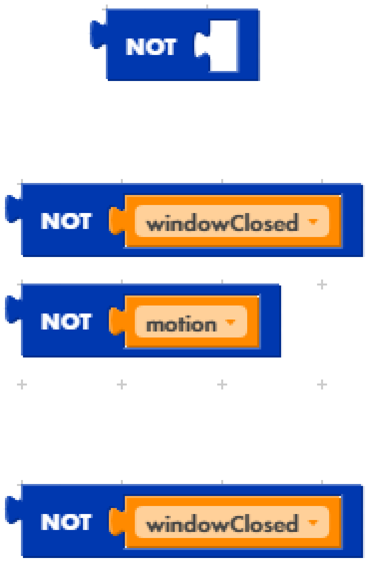




Test the Solution (Software)

$$X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ$$

1) Invert any inputs that have to be inverted using the inversion operator

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not windowClosed
not motion
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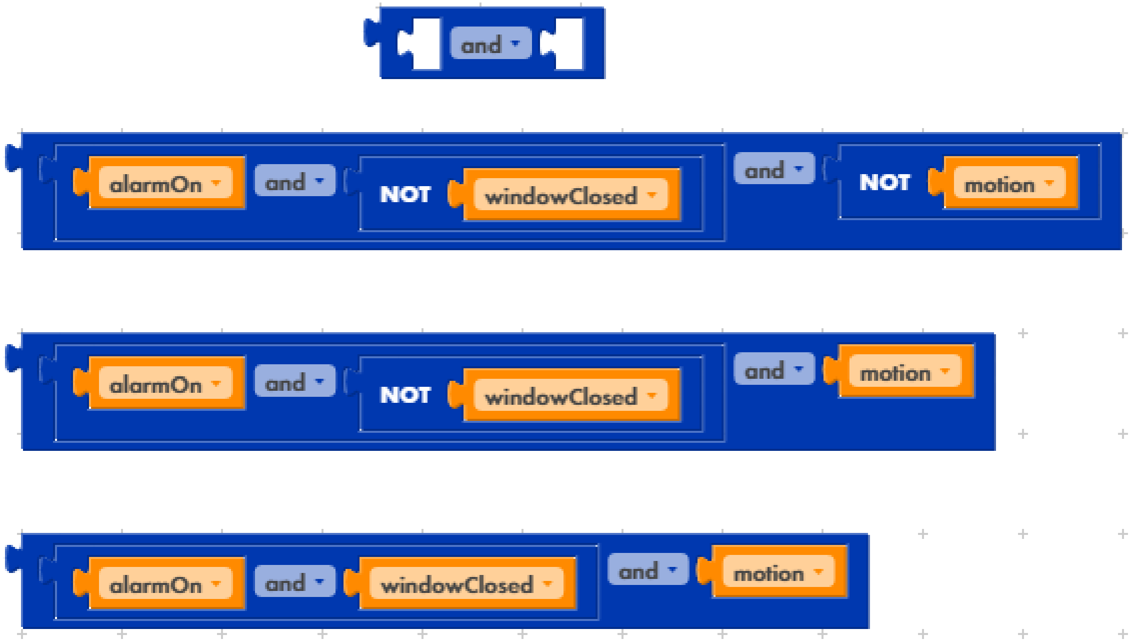
Test the Solution (Hardware)

$$X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ$$

2) AND the inputs to form a min-term using AND operators

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alarmOn and not windowClosed and not motion

alarmOn and not windowClosed and motion

alarmOn and windowClosed and motion
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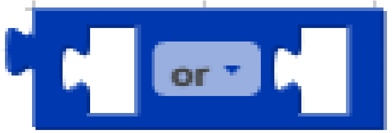
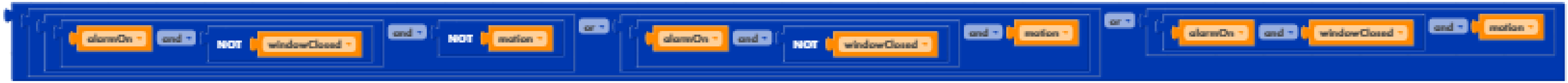
Test the Solution (Software)

$$X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ$$

3) OR the min-terms to form the solution using an OR Gate





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(alarmOn and not windowClosed and not motion) or (alarmOn and not windowClosed and motion) or (alarmOn and windowClosed and motion)
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References

Digital Logic Circuits created in National Instruments Multisim

Block-Based Code created in littleBits Code Kit

Text-Based Code created in Notepad++