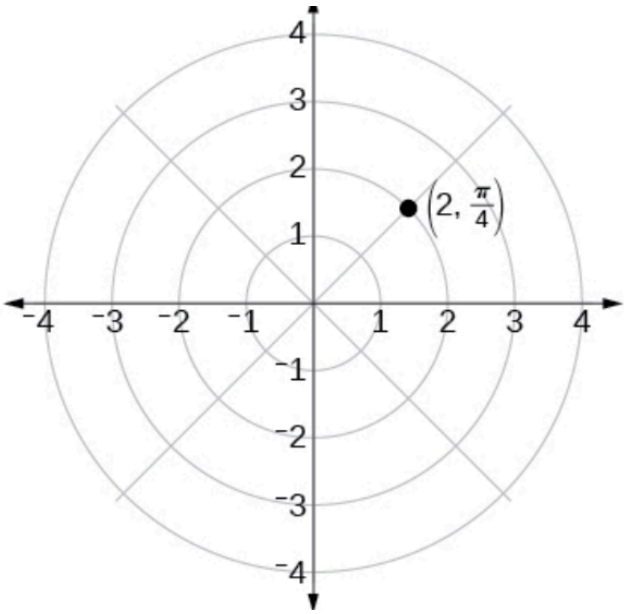
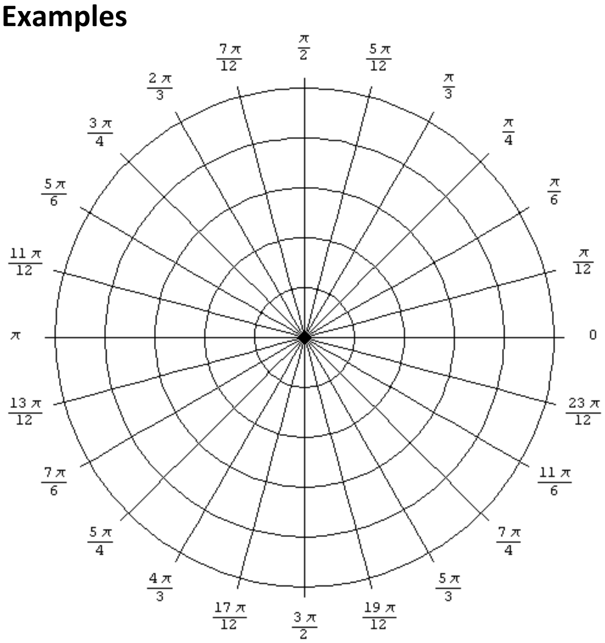


In this section, we introduce to polar coordinates, which are points labeled (,) and plotted on a polar grid. The polar grid is represented as a series of concentric circles radiating out from the pole, or the origin of the coordinate plane.

The polar grid is scaled as the unit circle with the positive ____-axis now viewed as the _____________ axis and the origin as the pole. The first coordinate r is the __________________ or length of the directed line segment from the pole. The angle ϑ, measured in radians, indicates the ____________________ of r. We move counterclockwise from the polar axis by an angle of ϑ, and measure a directed line segment the length of r in the direction of ϑ. Even though we measure ϑ first and then r, the polar point is written with the r-coordinate first.

Examples



Plot the point $\left(2, \frac{\pi}{3}\right)$ in the **polar grid**.

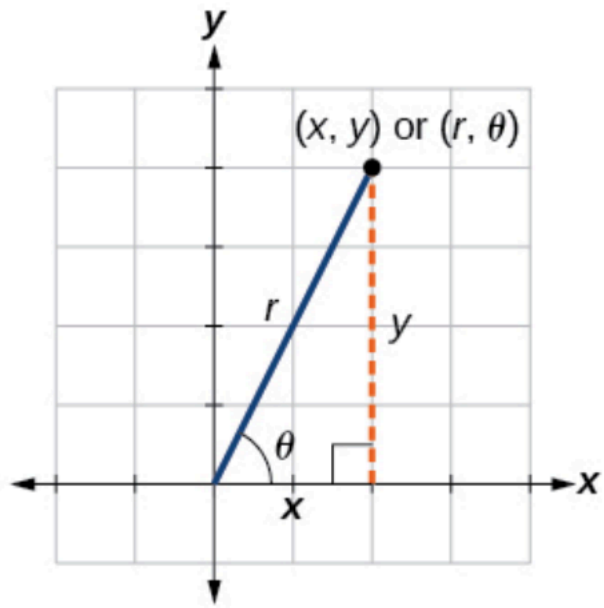
Plot the point $\left(-2, \frac{\pi}{6}\right)$ on the polar grid.

Plot the points $\left(3, -\frac{\pi}{6}\right)$and $\left(2, \frac{9\pi}{4}\right)$ on the same polar grid.

Converting from Polar Coordinates to Rectangular Coordinate

$\cos\,\theta = \frac{x}{r} \rightarrow x = r\cos\,\theta$

$\sin\,\theta = \frac{y}{r} \rightarrow y = r\sin\,\theta$



A GENERAL NOTE: CONVERTING FROM POLAR COORDINATES TO RECTANGULAR COORDINATES

To convert polar coordinates $(r,\ \theta)$ to rectangular coordinates $(x,\ y)$, let

$$\cos\,\theta = \frac{x}{r} \rightarrow x = r\cos\,\theta$$

$$\sin\,\theta = \frac{y}{r} \rightarrow y = r\sin\,\theta$$

HOW TO

Given polar coordinates, convert to rectangular coordinates.

1. Given the polar coordinate (r, θ), write $x = r\cos\,\theta$ and $y = r\sin\,\theta$.
2. Evaluate $\cos\,\theta$ and $\sin\,\theta$.
3. Multiply $\cos\,\theta$ by r to find the x-coordinate of the rectangular form.
4. Multiply $\sin\,\theta$ by r to find the y-coordinate of the rectangular form.

Examples

Write the polar coordinates $\left(3,\ \frac{\pi}{2}\right)$ as rectangular coordinates.

Write the polar coordinates $(-2, 0)$ as rectangular coordinates.

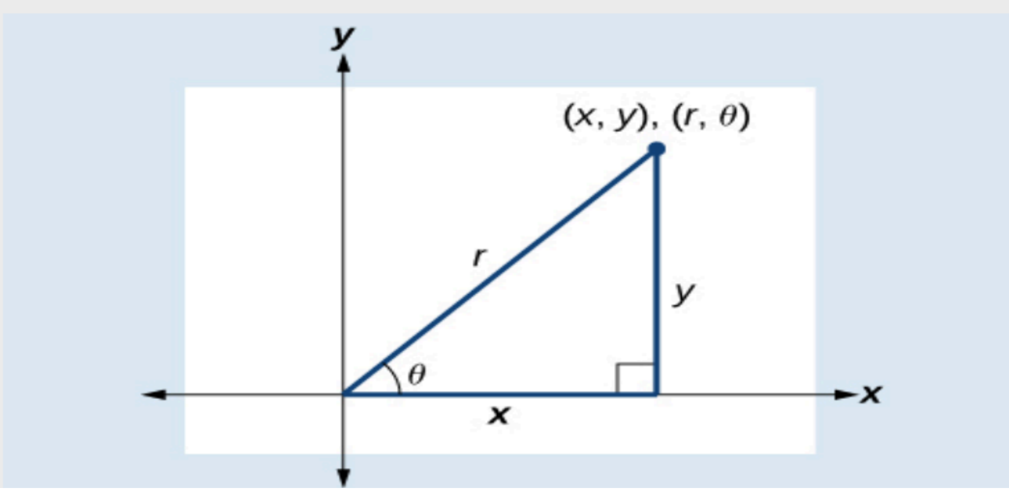
Write the polar coordinates $\left(-1, \frac{2\pi}{3}\right)$ as rectangular coordinates.

Converting from Rectangular Coordinates to Polar Coordinates

A GENERAL NOTE: CONVERTING FROM RECTANGULAR COORDINATES TO POLAR COORDINATES

Converting from rectangular coordinates to polar coordinates requires the use of one or more of the relationships illustrated in Figure.

$$\cos\theta = \frac{x}{r} \quad \text{or} \quad x = r\cos\theta$$
$$\sin\theta = \frac{y}{r} \quad \text{or} \quad y = r\sin\theta$$
$$r^2 = x^2 + y^2$$
$$\tan\theta = \frac{y}{x}$$



Example

Convert the rectangular coordinates $(3, 3)$ to polar coordinates.

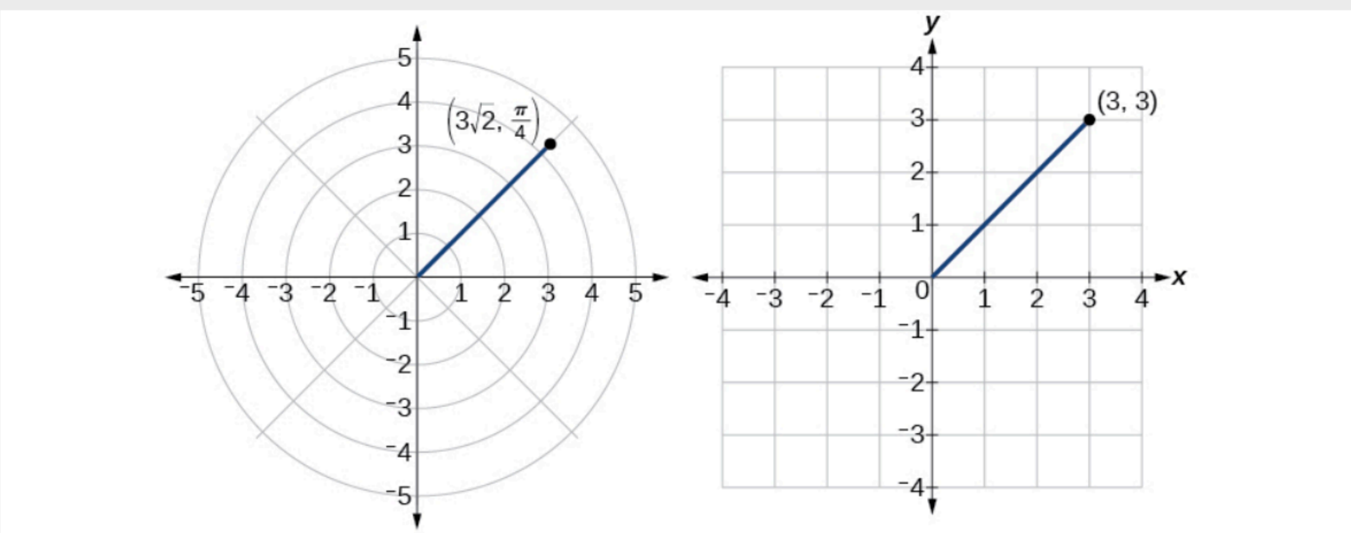


Figure 7.

Analysis

There are other sets of polar coordinates that will be the same as our first solution. For example, the points $\left(-3\sqrt{2}, \frac{5\pi}{4}\right)$ and $\left(3\sqrt{2}, -\frac{7\pi}{4}\right)$ will coincide with the original solution of $\left(3\sqrt{2}, \frac{\pi}{4}\right)$. The point $\left(-3\sqrt{2}, \frac{5\pi}{4}\right)$ indicates a move further counterclockwise by π, which is directly opposite $\frac{\pi}{4}$. The radius is expressed as $-3\sqrt{2}$. However, the angle $\frac{5\pi}{4}$ is located in the third quadrant and, as r is negative, we extend the directed line segment in the opposite direction, into the first quadrant. This is the same point as $\left(3\sqrt{2}, \frac{\pi}{4}\right)$. The point $\left(3\sqrt{2}, -\frac{7\pi}{4}\right)$ is a move further clockwise by $-\frac{7\pi}{4}$, from $\frac{\pi}{4}$. The radius, $3\sqrt{2}$, is the same.

Transforming Equations between Polar and Rectangular Forms

Examples

Write the Cartesian equation $x^2 + y^2 = 9$ in polar form.

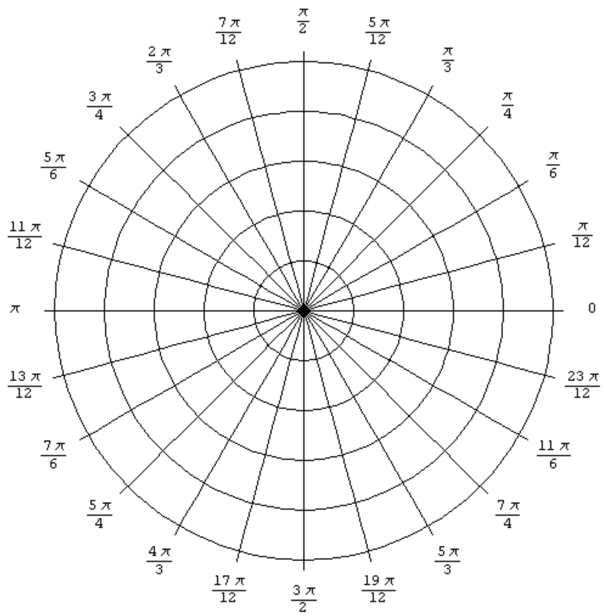
Rewrite the **Cartesian equation** $x^2 + y^2 = 6y$ as a polar equation.

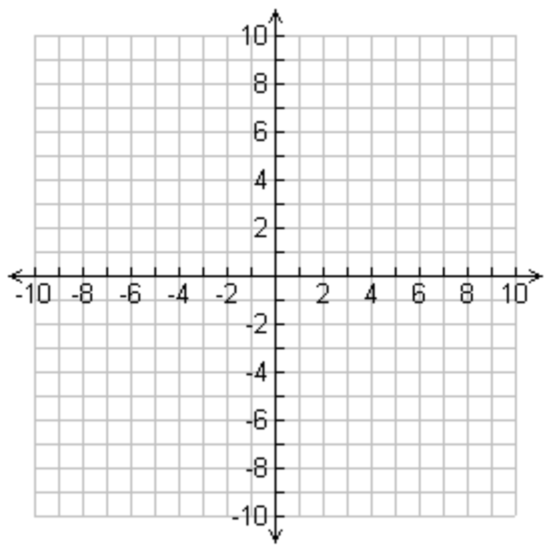
Rewrite the Cartesian equation $y = 3x + 2$ as a polar equation.

Identify and Graph Polar Equations by Converting to Rectangular Equations

Examples

Covert the polar equation $r = 2 \sec \theta$ to a rectangular equation, and draw its corresponding graph.





Rewrite the polar equation $r = \frac{3}{1 - 2\cos\theta}$ as a Cartesian equation.

Rewrite the polar equation $r = 2\sin\theta$ in Cartesian form.

Rewrite the polar equation $r = \sin(2\theta)$ in Cartesian form.