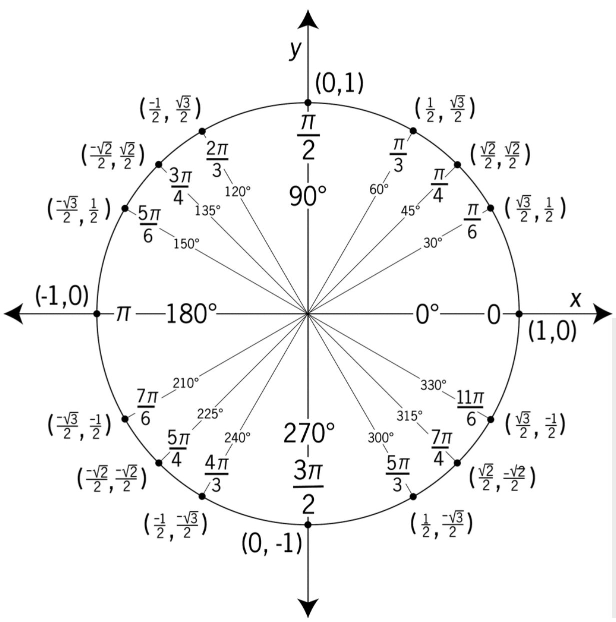
Solving Trigonometric Equations in Sine and Cosine

The period of both the sine function and the cosine function is 2π. In other words, every 2π units, the *y*-values repeat. If we need to find all possible solutions, then we must add __________,where k is an integer, to the initial solution.

Examples

Find all possible exact solutions for the equation $\cos\ \theta = \frac{1}{2}$.

Find all possible exact solutions for the equation $\sin\ t = \frac{1}{2}$.



HOW TO

Given a trigonometric equation, solve using algebra.

1. Look for a pattern that suggests an algebraic property, such as the difference of squares or a factoring opportunity.
2. Substitute the trigonometric expression with a single variable, such as x or u.
3. Solve the equation the same way an algebraic equation would be solved.
4. Substitute the trigonometric expression back in for the variable in the resulting expressions.
5. Solve for the angle.

Examples

Solve the equation exactly: $2\ \cos\ \theta - 3 = -5, 0 \le \theta < 2\pi$.

Solve exactly the following linear equation on the interval $[0, 2\pi)$: $2\ \sin\ x + 1 = 0$.

Solving Equations Involving Single Trigonometric Functions

We need to make several considerations when the equation involves trigonometric functions other than sine and cosine. Problems involving the reciprocals of the primary trigonometric functions need to be viewed from an algebraic perspective. In other words, we will write the reciprocal function, and solve for the angles using the function.

Examples

Solve the following equation exactly: $\csc\theta = -2, 0 \le \theta < 4\pi$.

Solve the equation exactly: $\tan\left(\theta - \frac{\pi}{2}\right) = 1, 0 \le \theta < 2\pi$.

Find all solutions for $\tan x = \sqrt{3}$.

Identify all exact solutions to the equation $2(\tan x + 3) = 5 + \tan x, 0 \le x < 2\pi$.

Solve Trigonometric Equations Using a Calculator

Not all functions can be solved exactly using only the unit circle. When we must solve an equation involving an angle other than one of the special angles, we will need to use a calculator. Make sure it is set to the proper mode, either degrees or radians, depending on the criteria of the given problem.

Examples

solve the equation $\sec\theta = -4$,giving your answer in radians.

Solve $\cos\theta = -0.2.$

Solving Trigonometric Equations in Quadratic Form

If there is only one function represented and one of the terms is squared, think about the standard form of a quadratic. Replace the trigonometric function with a variable such as x or u. If substitution makes the equation look like a quadratic equation, then we can use the same methods for solving quadratics to solve the trigonometric equations.

Examples

Solve the equation exactly: $\cos^2\theta + 3\cos\theta - 1 = 0, 0 \le \theta < 2\pi.$

Solve $\sin^2\theta = 2\cos\theta + 2, 0 \le \theta \le 2\pi.$ [Hint: Make a substitution to express the equation only in terms of cosine.]

$2\sin^2\theta + \sin\ \theta = 0; 0 \leq \theta < 2\pi$

Solve the quadratic equation $2\cos^2\theta + \cos\ \theta = 0.$

Solving Trigonometric Equations

Remember that the techniques we use for solving are not the same as those for verifying identities. The basic rules of algebra apply here, as opposed to rewriting one side of the identity to match the other side. In the next example, we use two identities to simplify the equation.

Examples

$$\cos\ x\ \cos(2x) + \sin\ x\ \sin(2x) = \frac{\sqrt{3}}{2}$$

Solve the equation exactly using an identity: $3\ \cos\ \theta + 3 = 2\sin^2\theta, 0 \leq \theta < 2\pi.$

Solving Trigonometric Equations with Multiple Angles

Sometimes it is not possible to solve a trigonometric equation with identities that have a multiple angle, such as $\sin(2x)$ or $\cos(3x)$. When confronted with these equations, recall that $y=\sin(2x)$ is a horizontal compression by a factor of 2 of the function $y=\sin x$. On an interval of 2π, we can graph two periods of $y=\sin(2x)$, as opposed to one cycle of $y=\sin x$. This compression of the graph leads us to believe there may be twice as many x-intercepts or solutions to $\sin(2x)=0$ compared to $\sin x=0$. This information will help us solve the equation.

Example

Solve exactly: $\cos(2x) = \frac{1}{2}$ on $[0, 2\pi)$.

Solving Right Triangle Problems

We can now use all of the methods we have learned to solve problems that involve applying the properties of right triangles and the Pythagorean Theorem. We begin with the familiar Pythagorean Theorem, $a^2+b^2=c^2$, and model an equation to fit a situation.

Example

OSHA safety regulations require that the base of a ladder be placed 1 foot from the wall for every 4 feet of ladder length. Find the angle that a ladder of any length forms with the ground and the height at which the ladder touches the wall.