9.1 - Solving Trigonometric Equations with Identities

Verifying the Fundamental Trigonometric Identities

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \cot^2\theta = \csc^2\theta \qquad 1 + \tan^2\theta = \sec^2\theta$$

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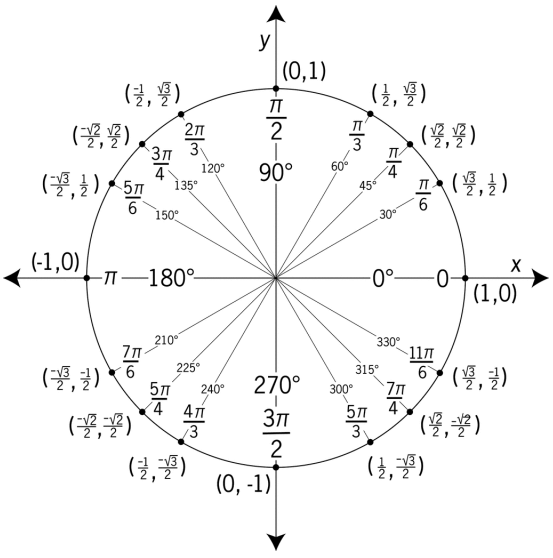
Prove: $1 + \cot^2\theta = \csc^2\theta$

Prove $1 + \tan^2\theta = \sec^2\theta$



Even-Odd Identities

$$\sin(-\theta) = -\sin\theta$$
$$\csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta$$
$$\sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta$$
$$\cot(-\theta) = -\cot\theta$$

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

HOW TO

Given a trigonometric identity, verify that it is true.

1. Work on one side of the equation. It is usually better to start with the more complex side, as it is easier to simplify than to build.
2. Look for opportunities to factor expressions, square a binomial, or add fractions.
3. Noting which functions are in the final expression, look for opportunities to use the identities and make the proper substitutions.
4. If these steps do not yield the desired result, try converting all terms to sines and cosines.

Examples

Verify $\tan\theta\cos\theta = \sin\theta$.

Verify the identity $\csc\theta\cos\theta\tan\theta = 1$.

Verify the following equivalency using the even-odd identities:

$$(1 + \sin x)[1 + \sin(-x)] = \cos^2 x$$

Verify the identity $\frac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

Show that $\frac{\cot\theta}{\csc\theta} = \cos\theta$.

Create an identity for the expression $2\tan\theta\sec\theta$ by rewriting strictly in terms of sine.

Verify

$$\frac{\sin^2(-\theta)-\cos^2(-\theta)}{\sin(-\theta)-\cos(-\theta)}=\cos\theta-\sin\theta$$

Verify

$$\frac{\sin^2\theta-1}{\tan\theta\sin\theta-\tan\theta}=\frac{\sin\theta+1}{\tan\theta}.$$

Verify the identity: $\left(1-\cos^2 x\right)\left(1+\cot^2 x\right)=1.$

Using Algebra (Factoring) to Simplify Trigonometric Expressions

Write the following trigonometric expression as an algebraic expression: $2\cos^2\theta + \cos\ \theta - 1.$

Rewrite the trigonometric expression using the difference of squares: $4\cos^2\theta - 1.$

Simplify the expression by rewriting and using identities:

$$\csc^2\theta - \cot^2\theta$$

Use algebraic techniques to verify the identity: $\frac{\cos\ \theta}{1+\sin\ \theta} = \frac{1-\sin\ \theta}{\cos\ \theta}.$