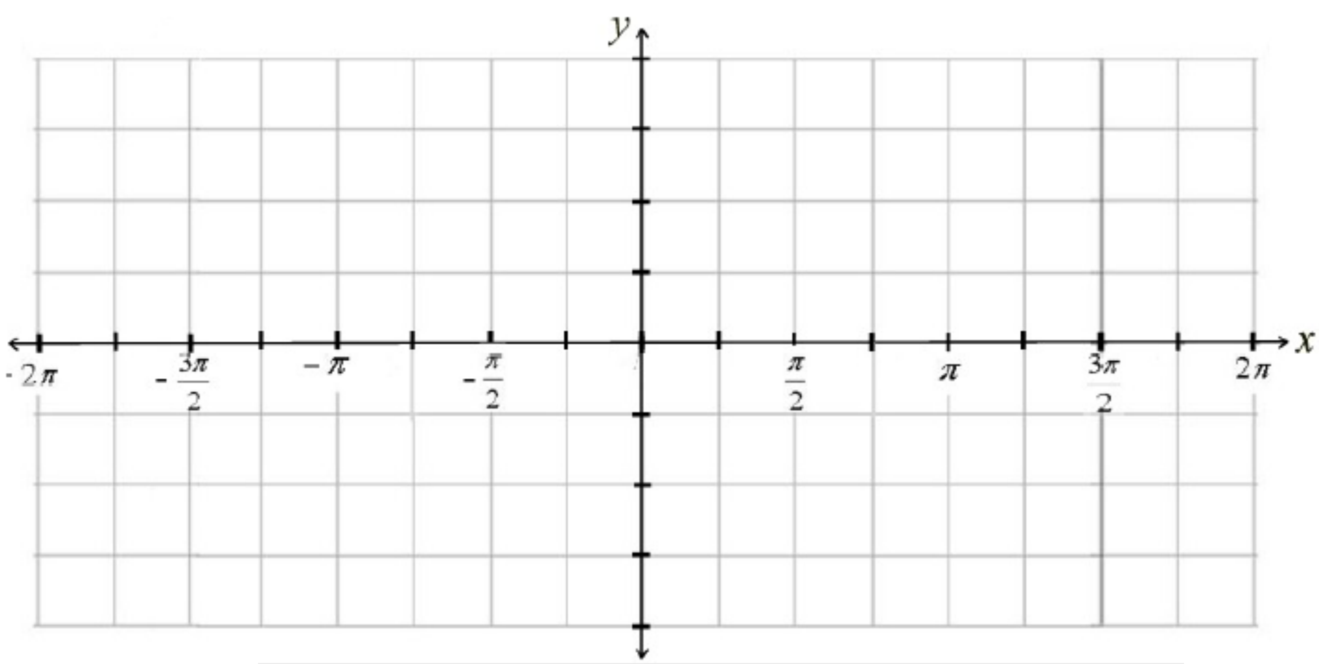
8.2 – Graphs and Other Trig Functions

Analyzing the Graph of $y = \tan x$

$$\tan x = \frac{\sin x}{\cos x}$$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
sin (x)	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cos (x)	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
tan (x)									



A GENERAL NOTE LABEL: FEATURES OF THE GRAPH OF $Y = A\text{TAN}(BX)$

- The stretching factor is $|A|$.
- The period is $P = \frac{\pi}{|B|}$.
- The domain is all real numbers x, where $x \neq \frac{\pi}{2|B|} + \frac{\pi}{|B|}k$ such that k is an integer.
- The range is $(-\infty, \infty)$.
- The asymptotes occur at $x = \frac{\pi}{2|B|} + \frac{\pi}{|B|}k$, where k is an integer.
- $y = A\tan(Bx)$ is an odd function.

Graphing Variations of $y = \tan x$

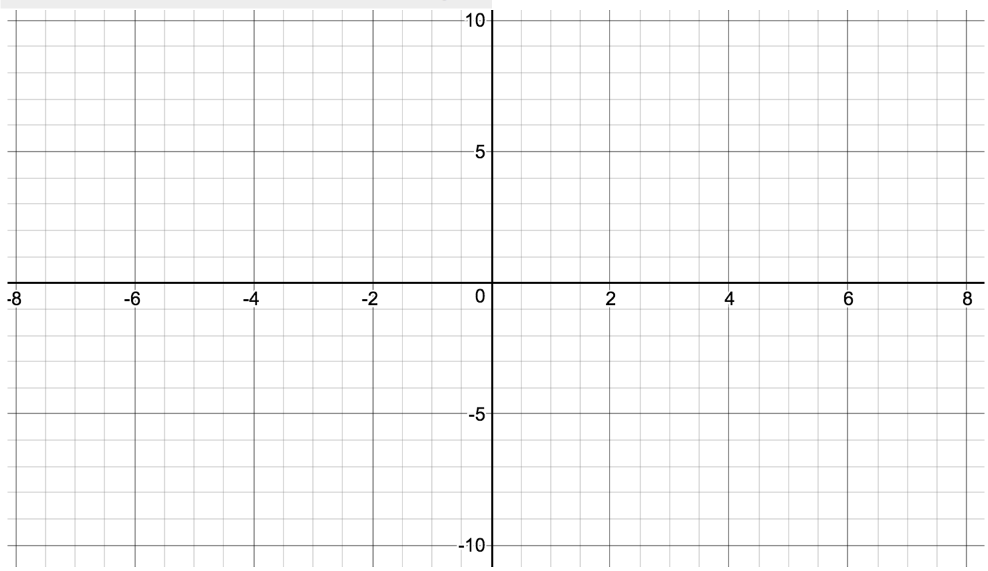
HOW TO FEATURE

Given the function $f(x) = A\tan(Bx)$, graph one period.

1. Identify the stretching factor, $|A|$.
2. Identify B and determine the period, $P = \frac{\pi}{|B|}$.
3. Draw vertical asymptotes at $x = -\frac{P}{2}$ and $x = \frac{P}{2}$.
4. For $A > 0$, the graph approaches the left asymptote at negative output values and the right asymptote at positive output values (reverse for $A < 0$).
5. Plot reference points at $\left(\frac{P}{4}, A\right)$, $(0, 0)$, and $\left(-\frac{P}{4}, -A\right)$, and draw the graph through these points.

Examples

Sketch a graph of $f(x) = 3\tan\left(\frac{\pi}{6}x\right)$



Graphing One Period of a Shifted Tangent Function

A GENERAL NOTE LABEL: FEATURES OF THE GRAPH OF $Y = A\tan(BX-C)+D$

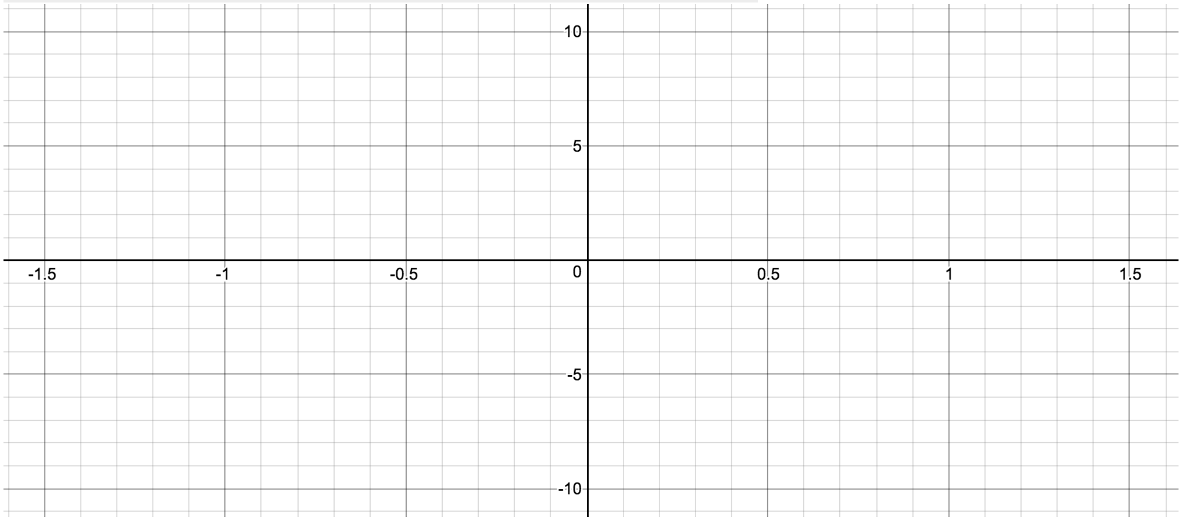
- The stretching factor is $|A|$.
- The period is $\frac{\pi}{|B|}$.
- The domain is $x \neq \frac{C}{B} + \frac{\pi}{|B|}k$, where k is an integer.
- The range is $(-\infty, -|A|] \cup [|A|, \infty)$.
- The vertical asymptotes occur at $x = \frac{C}{B} + \frac{\pi}{2|B|}k$, where k is an odd integer.
- There is no amplitude.
- $y = A\tan(Bx)$ is and odd function because it is the qoutient of odd and even functions(sin and cosine perspectively).

HOW TO FEATURE

Given the function $y = A\tan(Bx - C) + D$, sketch the graph of one period.

1. Express the function given in the form $y = A\tan(Bx - C) + D$.
2. Identify the **stretching/compressing factor**, $|A|$.
3. Identify B and determine the period, $P = \frac{\pi}{|B|}$.
4. Identify C and determine the phase shift, $\frac{C}{B}$.
5. Draw the graph of $y = A\tan(Bx)$ shifted to the right by $\frac{C}{B}$ and up by D.
6. Sketch the vertical asymptotes, which occur at $x = \frac{C}{B} + \frac{\pi}{2|B|}k$, where k is an odd integer.
7. Plot any three reference points and draw the graph through these points.

Graph one period of the function $y = -2\tan(\pi x + \pi) - 1$



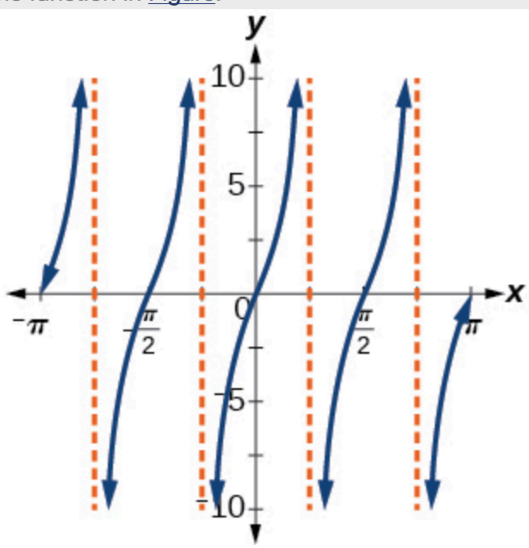
HOW TO FEATURE

Given the graph of a tangent function, identify horizontal and vertical stretches.

1. Find the period P from the spacing between successive vertical asymptotes or x-intercepts.
2. Write $f(x) = A\tan\left(\frac{\pi}{P}x\right)$.
3. Determine a convenient point $(x, f(x))$ on the given graph and use it to determine A.

Example

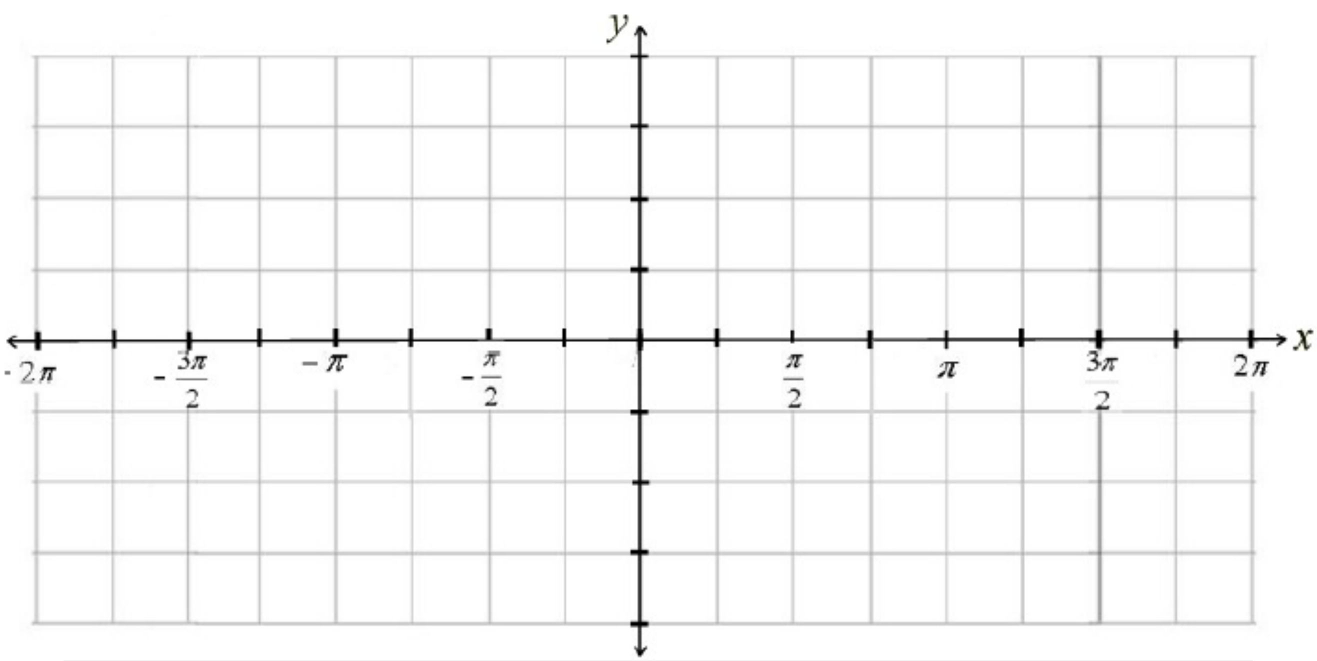
Find a formula for the function in Figure.



Analyzing the Graph of $y = \sec x$ and $y = \csc x$

$$\sec\ x = \frac{1}{\cos\ x}$$

x	*0*	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
cos (x)	*1*	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	*0*	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	*-1*
sec (x)									

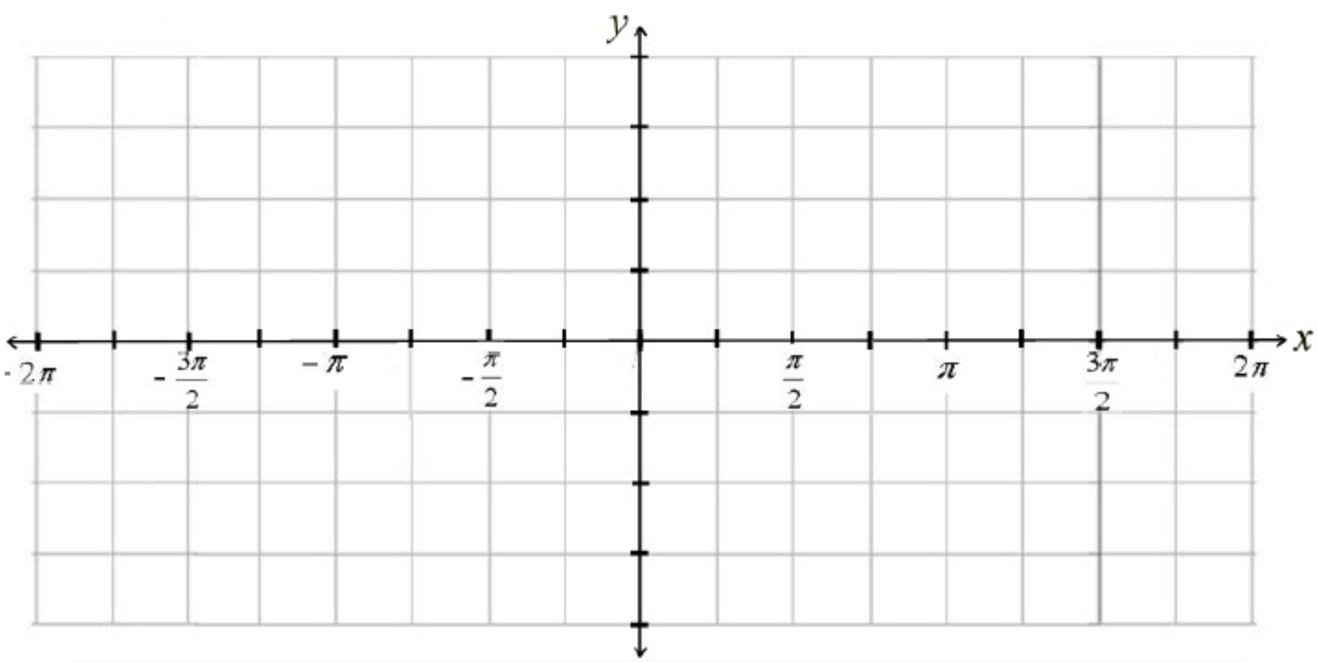


A GENERAL NOTE LABEL: FEATURES OF THE GRAPH OF $Y = ASEC(BX)$

- The stretching factor is $|A|$.
- The period is $\frac{2\pi}{|B|}$.
- The domain is $x \neq \frac{\pi}{2|B|}k,$ where k is an odd integer.
- The range is $(-\infty, -|A|] \cup [|A|, \infty)$.
- The vertical asymptotes occur at $x = \frac{\pi}{2|B|}k,$ where k is an odd integer.
- There is no amplitude.
- $y = A\sec(Bx)$ is an even function because cosine is an even function.

$\csc x = \frac{1}{\sin x}$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\csc(x)$									



A GENERAL NOTE LABEL: FEATURES OF THE GRAPH OF $Y = A\csc(BX)$

- The stretching factor is $|A|$.
- The period is $\frac{2\pi}{|B|}$.
- The domain is $x \neq \frac{\pi}{|B|}k$, where k is an integer.
- The range is $(-\infty, -|A|] \cup [|A|, \infty)$.
- The asymptotes occur at $x = \frac{\pi}{|B|}k$, where k is an integer.
- $y = A\csc(Bx)$ is an odd function because sine is an odd function.

Graphing Variations of $y = \sec x$

A GENERAL NOTE LABEL: FEATURES OF THE GRAPH OF $Y = A\sec(BX-C)+D$

- The stretching factor is $|A|$.
- The period is $\frac{2\pi}{|B|}$.
- The domain is $x \neq \frac{C}{B} + \frac{\pi}{2|B|}k$, where k is an odd integer.
- The range is $(-\infty, -|A|] \cup [|A|, \infty)$.
- The vertical asymptotes occur at $x = \frac{C}{B} + \frac{\pi}{2|B|}k$, where k is an odd integer.
- There is no amplitude.
- $y = A\sec(Bx)$ is an even function because cosine is an even function.

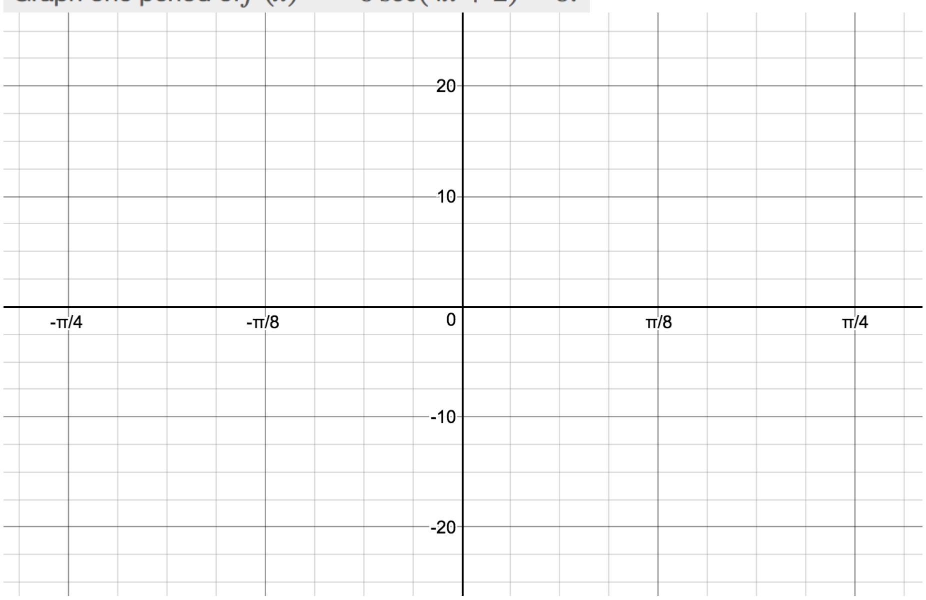
HOW TO FEATURE

Given a function of the form $f(x) = A\sec(Bx - C) + D$, graph one period.

1. Express the function given in the form $y = A\sec(Bx - C) + D$.
2. Identify the stretching/compressing factor, $|A|$.
3. Identify B and determine the period, $\frac{2\pi}{|B|}$.
4. Identify C and determine the phase shift, $\frac{C}{B}$.
5. Draw the graph of $y = A\sec(Bx)$ .but shift it to the right by $\frac{C}{B}$ and up by D.
6. Sketch the vertical asymptotes, which occur at $x = \frac{C}{B} + \frac{\pi}{2|B|}k$, where k is an odd integer.

Example

Graph one period of $f(x) = -6\sec(4x + 2) - 8$.



Graphing Variations of $y = \csc x$

A GENERAL NOTE LABEL: FEATURES OF THE GRAPH OF $Y = A\text{CSC}(BX-C)+D$

- The stretching factor is $|A|$.
- The period is $\frac{2\pi}{|B|}$.
- The domain is $x \neq \frac{C}{B} + \frac{\pi}{2|B|}k$, where k is an integer.
- The range is $(-\infty, -|A|] \cup [|A|, \infty)$.
- The vertical asymptotes occur at $x = \frac{C}{B} + \frac{\pi}{|B|}k$, where k is an integer.
- There is no amplitude.
- $y = A\csc(Bx)$ is an odd function because sine is an odd function.

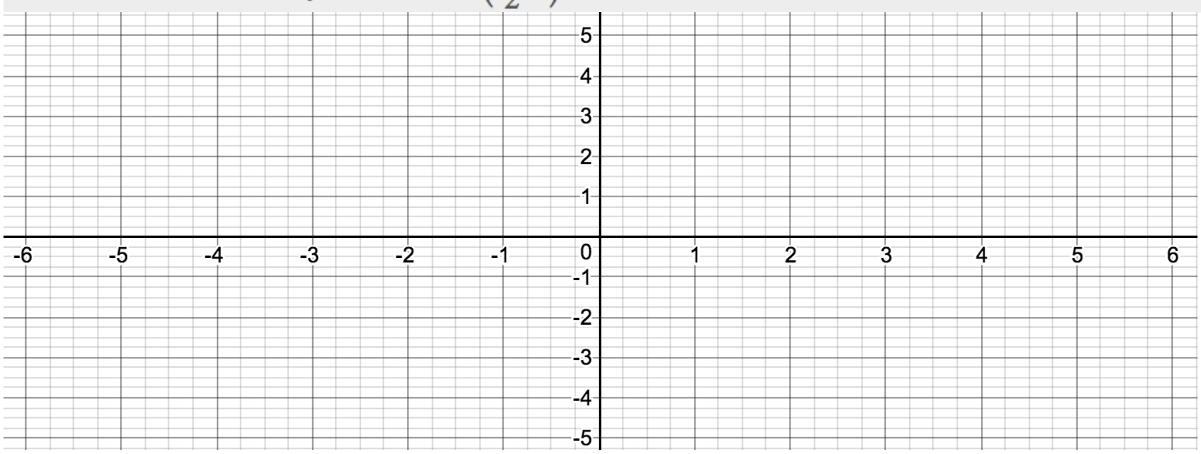
HOW TO FEATURE

Given a function of the form $f(x) = A\csc(Bx - C) + D$, graph one period.

1. Express the function given in the form $y = A\csc(Bx - C) + D$.
2. Identify the stretching/compressing factor, $|A|$.
3. Identify B and determine the period, $\frac{2\pi}{|B|}$.
4. Identify C and determine the phase shift, $\frac{C}{B}$.
5. Draw the graph of $y = A\csc(Bx)$ but shift it to the right by and up by D.
6. Sketch the vertical asymptotes, which occur at $x = \frac{C}{B} + \frac{\pi}{|B|}k$, where k is an integer.

Example

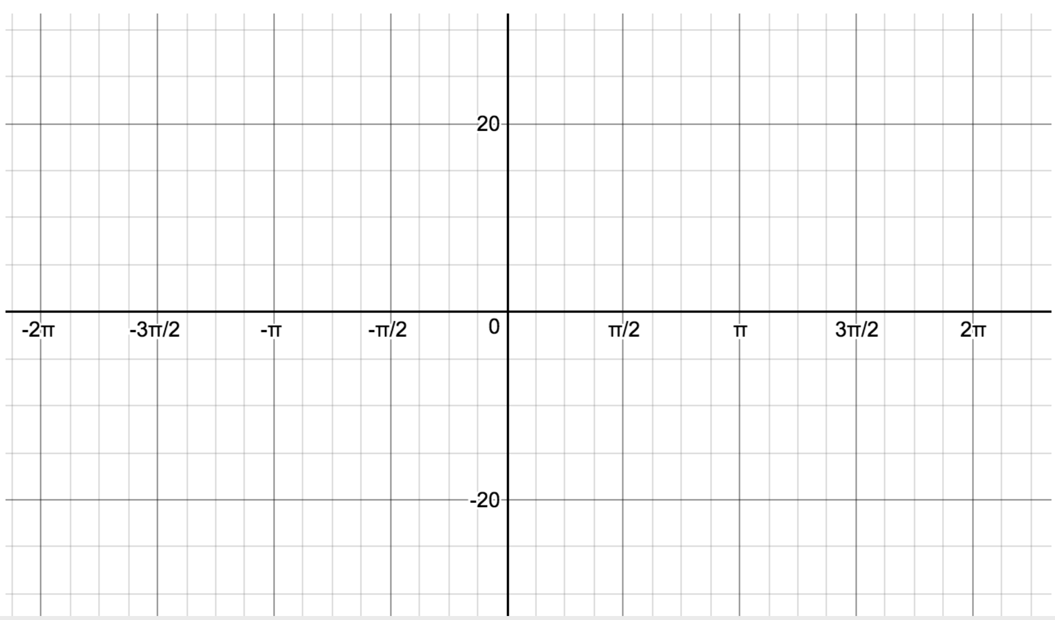
Sketch a graph of $y = 2\csc\left(\frac{\pi}{2}x\right) + 1$. What are the domain and range of this function?



Analyzing the Graph of $y = \cot x$

$\cot\ x = \frac{1}{\tan\ x}.$

x	*0*	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
cos (x)	*1*	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	*0*	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	*-1*
sin (x)	*0*	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	*1*	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	*0*
cot (x)									



A GENERAL NOTE LABEL: FEATURES OF THE GRAPH OF $Y = A\text{COT}(BX)$

- The stretching factor is $|A|$.
- The period is $P = \frac{\pi}{|B|}$.
- The domain is $x \neq \frac{\pi}{|B|}k,$ where k is an integer.
- The range is $(-\infty, \infty)$.
- The asymptotes occur at $x = \frac{\pi}{|B|}k,$ where k is an integer.
- $y = A\cot(Bx)$ is an odd function.

Graphing Variations of $y = \cot x$

A GENERAL NOTE LABEL: PROPERTIES OF THE GRAPH OF $Y = A\text{COT}(BX{-}C){+}D$

- The stretching factor is $|A|$.
- The period is $\frac{\pi}{|B|}$.
- The domain is $x \neq \frac{C}{B} + \frac{\pi}{|B|}k$, where k is an integer.
- The range is $(-\infty, -|A|] \cup [|A|, \infty)$.
- The vertical asymptotes occur at $x = \frac{C}{B} + \frac{\pi}{|B|}k$, where k is an integer.
- There is no amplitude.
- $y = A\cot(Bx)$ is an odd function because it is the quotient of even and odd functions (cosine and sine, respectively)

HOW TO FEATURE

Given a modified cotangent function of the form $f(x) = A\cot(Bx - C) + D$, graph one period.

1. Express the function in the form $f(x) = A\cot(Bx - C) + D$.
2. Identify the stretching factor, $|A|$.
3. Identify the period, $P = \frac{\pi}{|B|}$.
4. Identify the phase shift, $\frac{C}{B}$.
5. Draw the graph of $y = A\tan(Bx)$ shifted to the right by $\frac{C}{B}$ and up by D.
6. Sketch the asymptotes $x = \frac{C}{B} + \frac{\pi}{|B|}k$, where k is an integer.
7. Plot any three reference points and draw the graph through these points.

Example

Sketch a graph of one period of the function $f(x) = 4\cot\left(\frac{\pi}{8}x - \frac{\pi}{2}\right) - 2$.