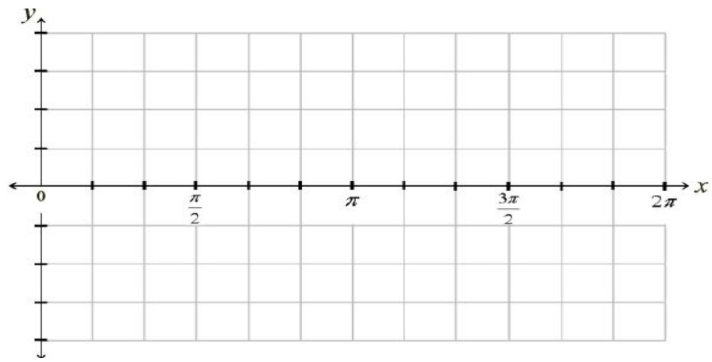
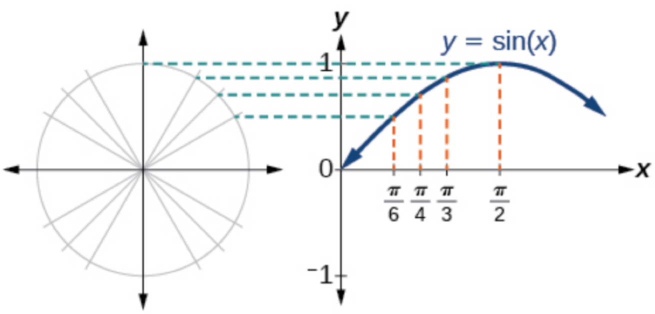
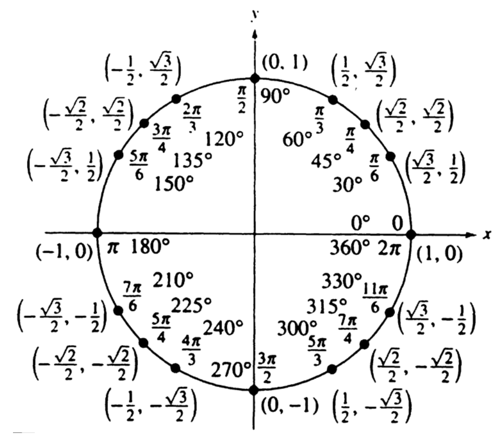
8.1 – Graphs of Sine and Cosine Functions

The Graph of $y = sin(x)$

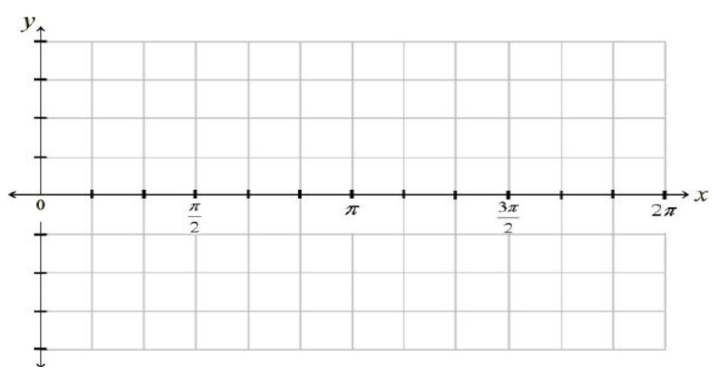


x	y



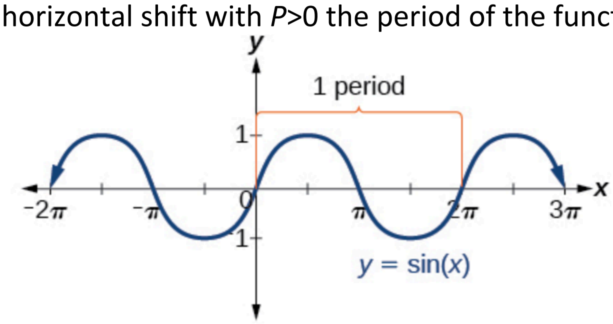


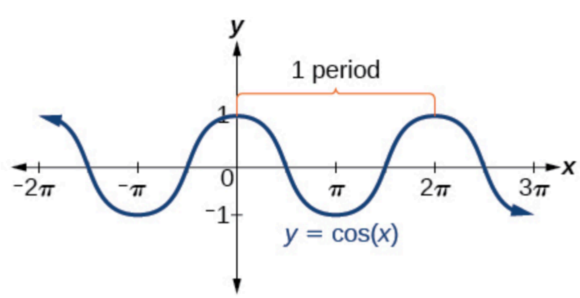
The graph of $y = cos(x)$



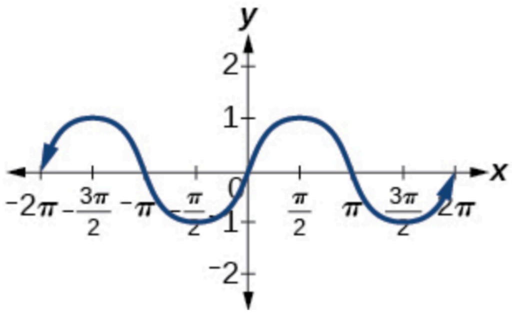
x	y

In both graphs, the shape of the graph repeats after 2π, which means the functions are periodic with a period of 2π. A periodic function is a function for which a specific horizontal shift, P, results in a function equal to the original function: $f(x+P)=f(x)$ for all values of x in the domain of f. When this occurs, we call the smallest such horizontal shift with $P>0$ the period of the function.

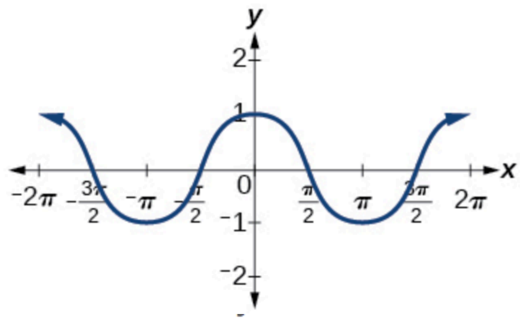




Even/Odd



$\sin(-x) = -\sin x.$



$\cos(-x) = \cos x.$

A GENERAL NOTE LABEL: CHARACTERISTICS OF SINE AND COSINE FUNCTIONS

The sine and cosine functions have several distinct characteristics:

- They are periodic functions with a period of 2π.
- The domain of each function is $(-\infty, \infty)$ and the range is $[-1, 1]$.
- The graph of $y = \sin x$ is symmetric about the origin, because it is an odd function.
- The graph of $y = \cos x$ is symmetric about the y-axis, because it is an even function.

Investigating Sinusoidal Functions

A function that has the same general shape as a sine or cosine function is known as a sinusoidal function. The general forms of sinusoidal functions are

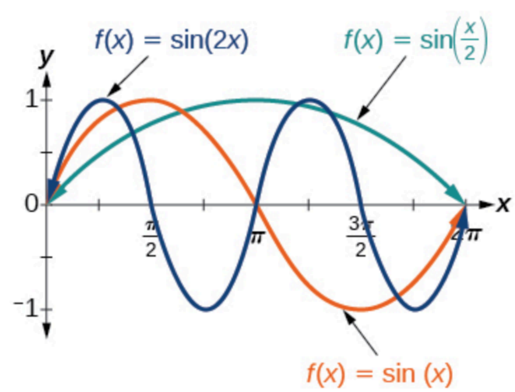
$$y = A\sin(Bx - C) + D$$

and

$$y = A\cos(Bx - C) + D$$

Determining the Period of Sinusoidal Functions

In the general formula, B is related to the period by $P = \frac{2\pi}{|B|}$.



A GENERAL NOTE LABEL: PERIOD OF SINUSOIDAL FUNCTIONS

If we let $C = 0$ and $D = 0$ in the general form equations of the sine and cosine functions, we obtain the forms

$$y = A\sin(Bx)$$

$$y = A\cos(Bx)$$

The period is $\frac{2\pi}{|B|}$.

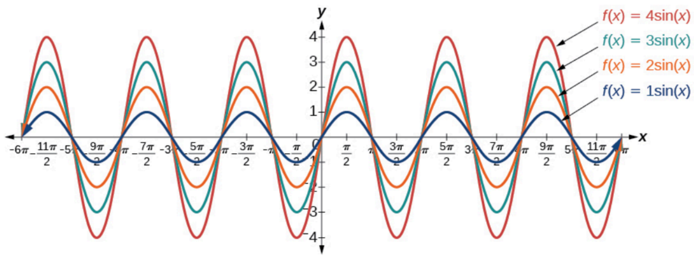
Examples

Determine the period of the function $f(x) = \sin\left(\frac{\pi}{6}x\right)$.

Determine the period of the function $g(x) = \cos\left(\frac{x}{3}\right)$

Determining Amplitude

Now let's turn to the variable *A,* so we can analyze how it is related to the _____________________, or greatest distance from rest. *A* represents the vertical stretch factor, and its absolute value |*A*| is the amplitude. The local maxima will be a distance |*A*| above the vertical **midline** of the graph, which is the line *x*=*D*; because *D*=0 in this case, the midline is the *x*-axis. The local minima will be the same distance below the midline. If |*A*|>1,the function is stretched.



A GENERAL NOTE LABEL: AMPLITUDE OF SINUSOIDAL FUNCTIONS

If we let $C = 0$ and $D = 0$ in the general form equations of the sine and cosine functions, we obtain the forms

$$y = A\sin(Bx) \text{ and } y = A\cos(Bx)$$

The **amplitude** is A, and the vertical height from the **midline** is $|A|$. In addition, notice in the example that

$$|A| = \text{amplitude} = \frac{1}{2}|\text{maximum} - \text{minimum}|$$

Examples

What is the amplitude of the sinusoidal function $f(x) = -4\sin(x)$? Is the function stretched or compressed vertically?

What is the amplitude of the sinusoidal function $f(x) = \frac{1}{2}\sin(x)$? Is the function stretched or compressed vertically?

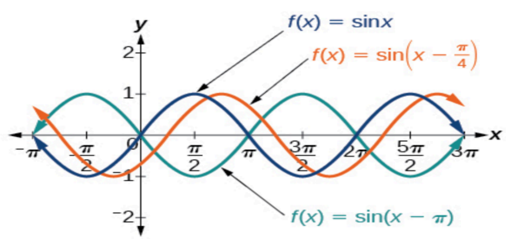
Analyzing Graphs of Variations of $y = \sin x$ and $y = \cos x$

$$y = A\sin(Bx - C) + D \text{ and } y = A\cos(Bx - C) + D$$

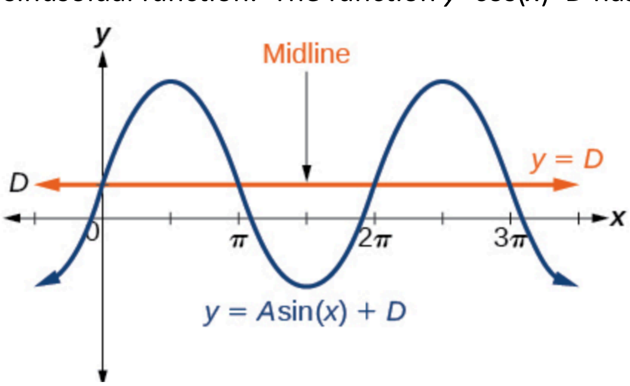
or

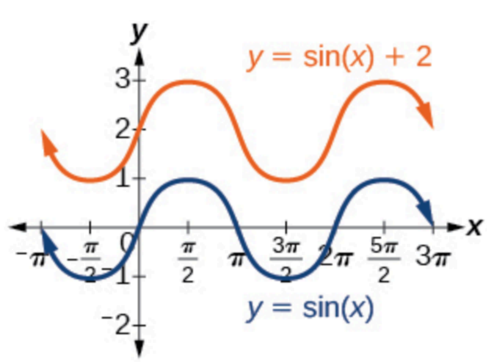
$$y = A\sin\left(B\left(x - \frac{C}{B}\right)\right) + D \text{ and } y = A\cos\left(B\left(x - \frac{C}{B}\right)\right) + D$$

The value $\frac{C}{B}$ for a sinusoidal function is called the ______________ ________________, or the horizontal displacement of the basic sine or cosine function. If *C*>0, the graph shifts to the __________. If *C*<0, the graph shifts to the _________. The greater the value of|*C*|,the more the graph is shifted. The figure below shows that the graph of *f*(*x*)=sin(*x*−*π*)shifts to the _______ by ___ units, which is more than we see in the graph of *f*(*x*)=sin($x - \frac{\pi}{4}$),which shifts to the right by _____units.



While *C* relates to the horizontal shift, *D* indicates the vertical shift from the midline in the general formula for a sinusoidal function. The function *y*=cos(*x*)+*D* has its midline at *y*=*D*.





A GENERAL NOTE LABEL: VARIATIONS OF SINE AND COSINE FUNCTIONS

Given an equation in the form $f(x) = A\sin(Bx - C) + D$ or $f(x) = A\cos(Bx - C) + D$, $\frac{C}{B}$ is the **phase shift** and D is the **vertical shift**.

Examples

Determine the direction and magnitude of the phase shift for $f(x) = 3\cos\left(x - \frac{\pi}{2}\right)$

Determine the direction and magnitude of the vertical shift for $f(x) = 3\sin(x) + 2$

HOW TO FEATURE

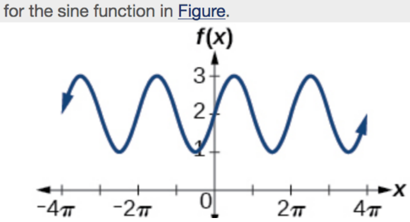
Given a sinusoidal function in the form $f(x) = A\sin(Bx - C) + D$, identify the midline, amplitude, period, and phase shift.

1. Determine the amplitude as $|A|$.
2. Determine the period as $P = \frac{2\pi}{|B|}$.
3. Determine the phase shift as $\frac{C}{B}$.
4. Determine the midline as $y = D$.

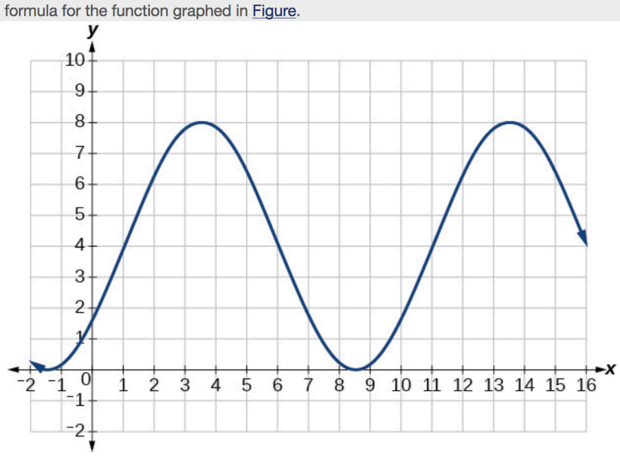
Examples

Determine the midline, amplitude, period, and phase shift of the function $y = \frac{1}{2}\cos\left(\frac{x}{3} - \frac{\pi}{3}\right)$

Determine the formula for the sine function in Figure.



Write a formula for the function graphed in Figure.



Graphing Variations of $y = \sin x$ and $y = \cos x$

HOW TO FEATURE

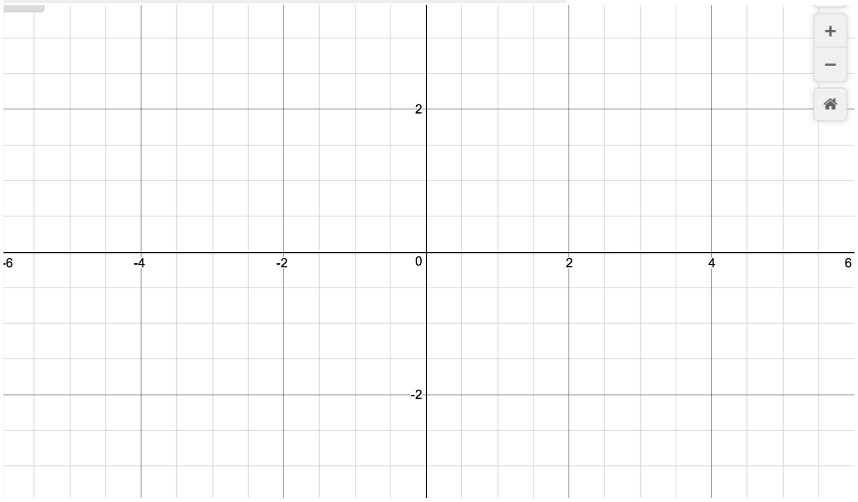
Given the function $y = A\sin(Bx)$, sketch its graph.

1. Identify the amplitude, $|A|$.
2. Identify the period, $P = \frac{2\pi}{|B|}$.
3. Start at the origin, with the function increasing to the right if A is positive or decreasing if A is negative.
4. At $x = \frac{\pi}{2|B|}$ there is a local maximum for $A > 0$ or a minimum for $A < 0$, with $y = A$.
5. The curve returns to the x-axis at $x = \frac{\pi}{|B|}$.
6. There is a local minimum for $A > 0$ (maximum for $A < 0$) at $x = \frac{3\pi}{2|B|}$ with $y = -A$.
7. The curve returns again to the x-axis at $x = \frac{\pi}{2|B|}$.

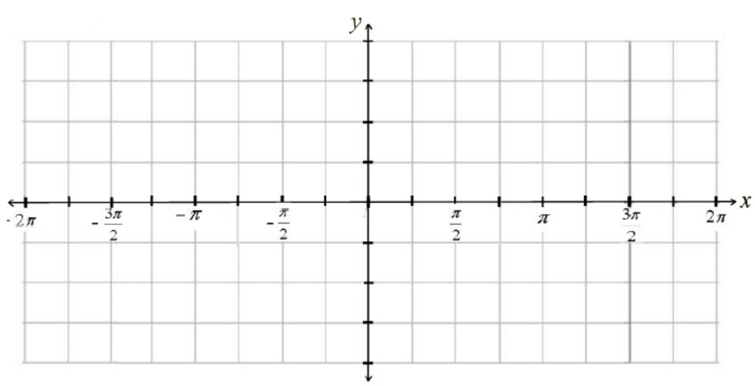
Examples

Graphing a Function and Identifying the Amplitude and Period

Sketch a graph of $f(x) = -2\sin\left(\frac{\pi x}{2}\right)$.



Sketch a graph of $g(x) = -0.8\cos(2x)$. Determine the midline, amplitude, period, and phase shift.



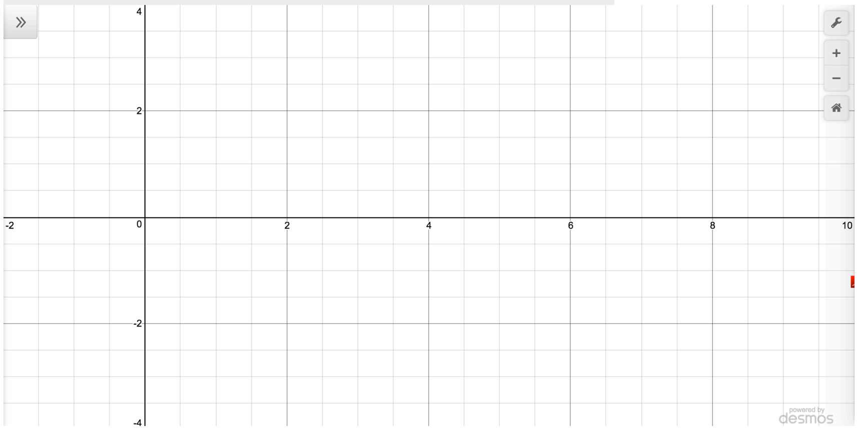
HOW TO FEATURE

Given a sinusoidal function with a phase shift and a vertical shift, sketch its graph.

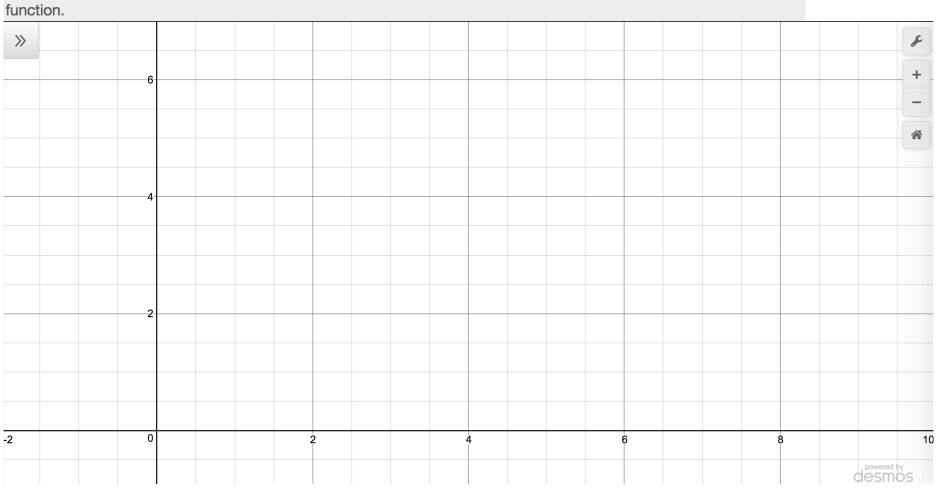
1. Express the function in the general form $y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$.
2. Identify the amplitude, $|A|$.
3. Identify the period, $P = \frac{2\pi}{|B|}$.
4. Identify the phase shift, $\frac{C}{B}$.
5. Draw the graph of $f(x) = A\sin(Bx)$ shifted to the right or left by $\frac{C}{B}$ and up or down by D.

Examples

Sketch a graph of $f(x) = 3\sin\left(\frac{\pi}{4}x - \frac{\pi}{4}\right)$



Given $y = -2\cos\left(\frac{\pi}{2}x + \pi\right) + 3$, determine the amplitude, period, phase shift, and horizontal shift. Then graph the function.

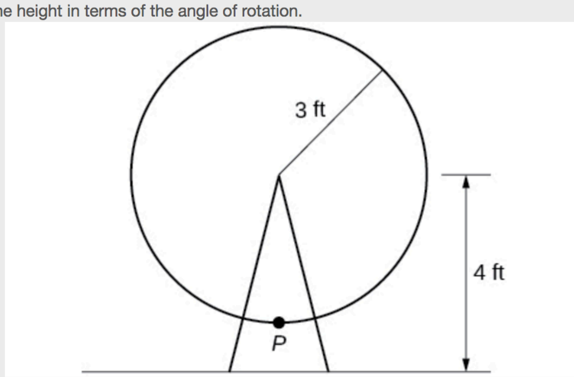


Using Transformations of Sine and Cosine Functions

We can use the transformations of sine and cosine functions in numerous applications. As mentioned at the beginning of the chapter, circular motion can be modeled using either the sine or cosine function.

Examples

A circle with radius 3 ft is mounted with its center 4 ft off the ground. The point closest to the ground is labeled *P*, as shown in Figure. Sketch a graph of the height above the ground of the point P as the circle is rotated; then find a function that gives the height in terms of the angle of rotation.



A weight is attached to a spring that is then hung from a board, as shown in Figure. As the spring oscillates up and down, the position y of the weight relative to the board ranges from -1 in. (at time $x = 0$) to -7 in. (at time $x = \pi$) below the board. Assume the position of y is given as a sinusoidal function of x. Sketch a graph of the function, and then find a cosine function that gives the position y in terms of x.