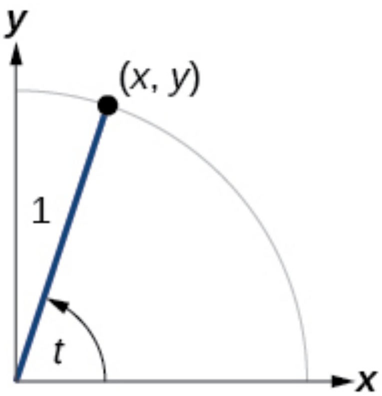
7.4 – The Other Trig Functions

Finding Exact Values

As with the sine and cosine, we can use the (x,y) coordinates to find the other functions.



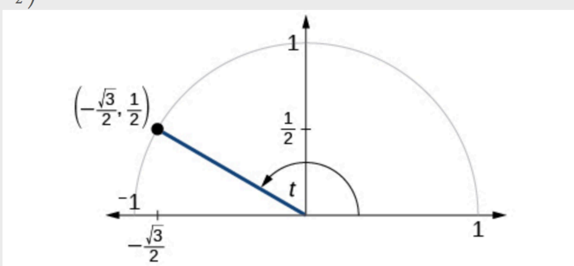
A GENERAL NOTE: TANGENT, SECANT, COSECANT, AND COTANGENT FUNCTIONS

If t is a real number and (x, y) is a point where the terminal side of an angle of t radians intercepts the unit circle, then

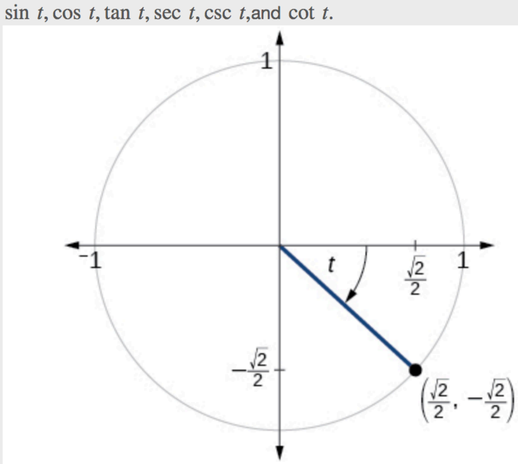
$$\begin{aligned} \tan t &= \frac{y}{x}, x \neq 0 \\ \sec t &= \frac{1}{x}, x \neq 0 \\ \csc t &= \frac{1}{y}, y \neq 0 \\ \cot t &= \frac{x}{y}, y \neq 0 \end{aligned}$$

Examples

The point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ is on the unit circle, as shown in Figure. Find $\sin t, \cos t, \tan t, \sec t, \csc t,$and $\cot t$.



The point $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ is on the unit circle, as shown in Figure.
Find $\sin t, \cos t, \tan t, \sec t, \csc t,$and $\cot t$.

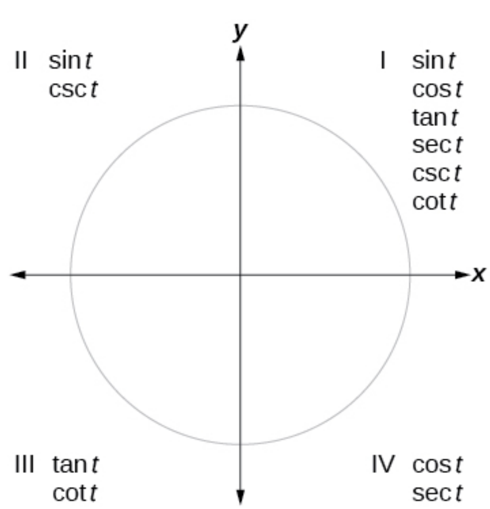


Angle	0	$\frac{\pi}{6}$, or 30°	$\frac{\pi}{4}$, or 45°	$\frac{\pi}{3}$, or 60°	$\frac{\pi}{2}$, or 90°
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Tangent	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined
Secant	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	Undefined
Cosecant	Undefined	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
Cotangent	Undefined	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

Example

Find $\sin\ t, \cos\ t, \tan\ t, \sec\ t, \csc\ t,$and $\cot\ t.$ when $t = \frac{\pi}{3}.$

Using Reference Angles to Evaluate Secant, Cosecant, and Cotangent



We can evaluate trigonometric functions of angles outside the first quadrant using reference angles as we have already done with the sine and cosine functions.

A Smart Trig Class

To help remember which of the six trigonometric functions are positive in each quadrant, we can use the mnemonic phrase "A Smart Trig Class." Each of the four words in the phrase corresponds to one of the four quadrants, starting with quadrant I and rotating counterclockwise. In quadrant I, which is "**A**," **a**ll of the six trigonometric functions are positive. In quadrant II, "**S**mart," only **s**ine and its reciprocal function, cosecant, are positive. In quadrant III, "**T**rig," only **t**angent and its reciprocal function, cotangent, are positive. Finally, in quadrant IV, "**C**lass," only **c**osine and its reciprocal function, secant, are positive.

HOW TO

Given an angle not in the first quadrant, use reference angles to find all six trigonometric functions.

1. Measure the angle formed by the terminal side of the given angle and the horizontal axis. This is the reference angle.
2. Evaluate the function at the reference angle.
3. Observe the quadrant where the terminal side of the original angle is located. Based on the quadrant, determine whether the output is positive or negative.

Examples

Use reference angles to find all six trigonometric functions of $-\frac{7\pi}{4}$.

Using Even and Odd Trig Functions

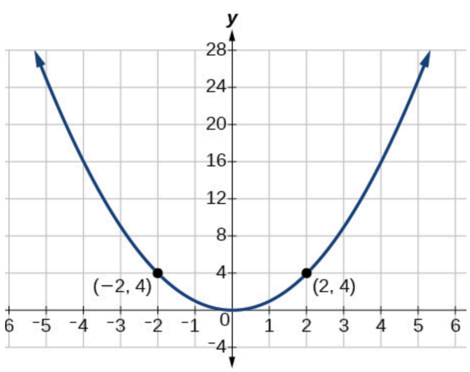


Figure 5. The function $f(x) = x^2$ is an even function.

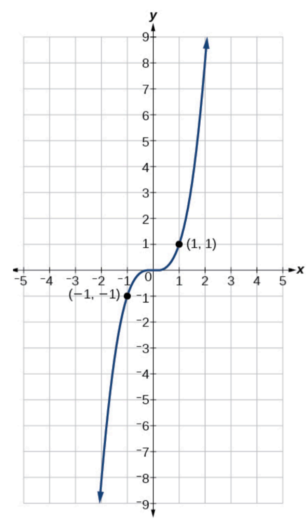
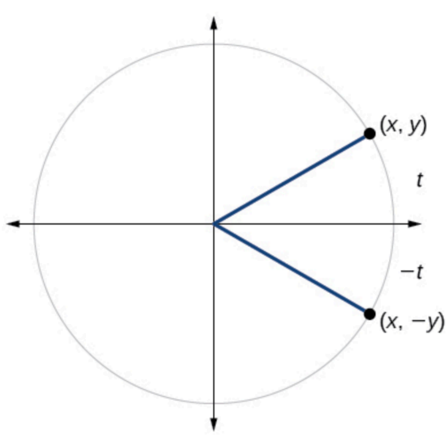
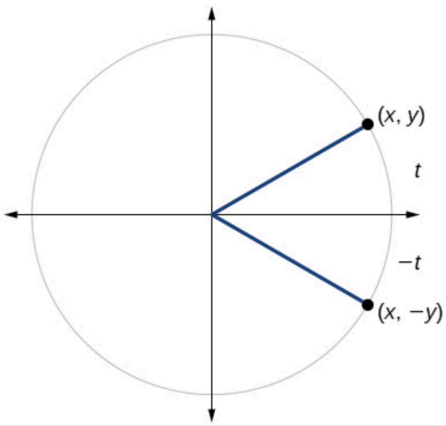


Figure 6. The function $f(x) = x^3$ is an odd function.



We can test whether a trigonometric function is even or odd by drawing a unit circle with a positive and a negative angle, as in [Figure](). The sine of the positive angle is y. The sine of the negative angle is $-y$. The sine function, then, is an odd function. We can test each of the six trigonometric functions in this fashion.



$\sin t = y$	$\cos t = x$	$\tan(t) = \frac{y}{x}$
$\sin(-t) = -y$	$\cos(-t) = x$	$\tan(-t) = -\frac{y}{x}$
$\sin t \neq \sin(-t)$	$\cos t = \cos(-t)$	$\tan t \neq \tan(-t)$
$\sec t = \frac{1}{x}$	$\csc t = \frac{1}{y}$	$\cot t = \frac{x}{y}$
$\sec(-t) = \frac{1}{x}$	$\csc(-t) = \frac{1}{-y}$	$\cot(-t) = \frac{x}{-y}$
$\sec t = \sec(-t)$	$\csc t \neq \csc(-t)$	$\cot t \neq \cot(-t)$

A GENERAL NOTE: EVEN AND ODD TRIGONOMETRIC FUNCTIONS

An even function is one in which $f(-x) = f(x)$.

An odd function is one in which $f(-x) = -f(x)$.

Cosine and secant are even:

$$\begin{aligned} \cos(-t) &= \cos t \\ \sec(-t) &= \sec t \end{aligned}$$

Sine, tangent, cosecant, and cotangent are odd:

$$\begin{aligned} \sin(-t) &= -\sin t \\ \tan(-t) &= -\tan t \\ \csc(-t) &= -\csc t \\ \cot(-t) &= -\cot t \end{aligned}$$

Examples

If the secant of angle t is 2, what is the secant of $-t$?

If the cotangent of angle t is $\sqrt{3}$,what is the cotangent of $-t$?

Recognizing and Using Fundamental Identities

A GENERAL NOTER: FUNDAMENTAL IDENTITIES

We can derive some useful **identities** from the six trigonometric functions. The other four trigonometric functions can be related back to the sine and cosine functions using these basic relationships:

$$\tan t = \frac{\sin t}{\cos t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\cot t = \frac{1}{\tan t} = \frac{\cos t}{\sin t}$$

Examples

,evaluate $\tan(45°)$.

Evaluate $\csc\left(\frac{7\pi}{6}\right)$.

,evaluate $\sec\left(\frac{5\pi}{6}\right)$.

Simplify $\frac{\sec t}{\tan t}$.

Simplify $\tan t(\cos t)$.

Alternate forms of the Pythagorean Theorem

$$\frac{\cos^2 t}{\cos^2 t} + \frac{\sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$$

$$\frac{\cos^2 t}{\sin^2 t} + \frac{\sin^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}$$

A GENERAL NOTE: ALTERNATE FORMS OF THE PYTHAGOREAN IDENTITY

$$1 + \tan^2 t = \sec^2 t$$

$$\cot^2 t + 1 = \csc^2 t$$

Example

If $\sec(t) = -\frac{17}{8}$ and $0 < t < \pi$, find the values of the other five functions.

As we discussed at the beginning of the chapter, a function that repeats its values in regular intervals is known as a periodic function. The trigonometric functions are periodic. For the four trigonometric functions, sine, cosine, cosecant and secant, a revolution of one circle, or 2π, will result in the same outputs for these functions. And for tangent and cotangent, only a half a revolution will result in the same outputs. A ___________________________ is the shortest interval over which a function completes one full cycle.

A GENERAL NOTE: PERIOD OF A FUNCTION

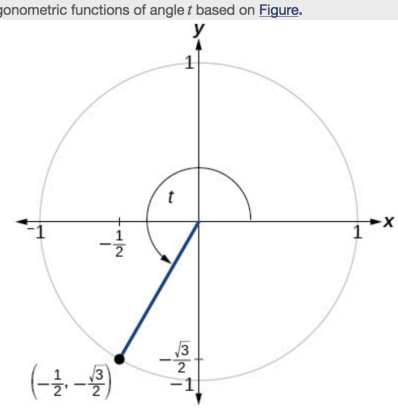
The **period** P of a repeating function f is the number representing the interval such that $f(x+P) = f(x)$ for any value of x.

The period of the cosine, sine, secant, and cosecant functions is 2π.

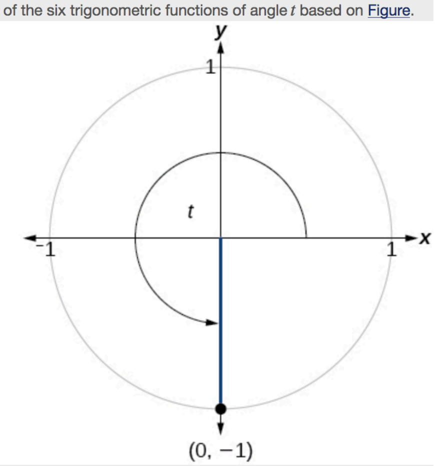
The period of the tangent and cotangent functions is π.

Examples

Find the values of the six trigonometric functions of angle t based on Figure.



Find the values of the six trigonometric functions of angle t based on Figure.



$\sin(t) = \frac{\sqrt{2}}{2}$ and $\cos(t) = \frac{\sqrt{2}}{2}$, find $\sec(t)$, $\csc(t)$, $\tan(t)$, and $\cot(t)$

“”

Evaluating Trig Functions with a Calculator

Evaluating a tangent function with a scientific calculator as opposed to a graphing calculator or computer algebra system is like evaluating a sine or cosine: Enter the value and press the TAN key. For the reciprocal functions, there may not be any dedicated keys that say CSC, SEC, or COT. In that case, the function must be evaluated as the reciprocal of a sine, cosine, or tangent.

HOW TO

Given an angle measure in radians, use a scientific calculator to find the cosecant.

1. If the calculator has degree mode and radian mode, set it to radian mode.
2. Enter: 1/
3. Enter the value of the angle inside parentheses.
4. Press the SIN key.
5. Press the = key.

HOW TO

Given an angle measure in radians, use a graphing utility/calculator to find the cosecant.

- If the graphing utility has degree mode and radian mode, set it to radian mode.
- Enter: 1/
- Press the SIN key.
- Enter the value of the angle inside parentheses.
- Press the ENTER key.

Examples

Evaluate the cosecant of $\frac{5\pi}{7}$.

Evaluate the cotangent of $-\frac{\pi}{8}$.