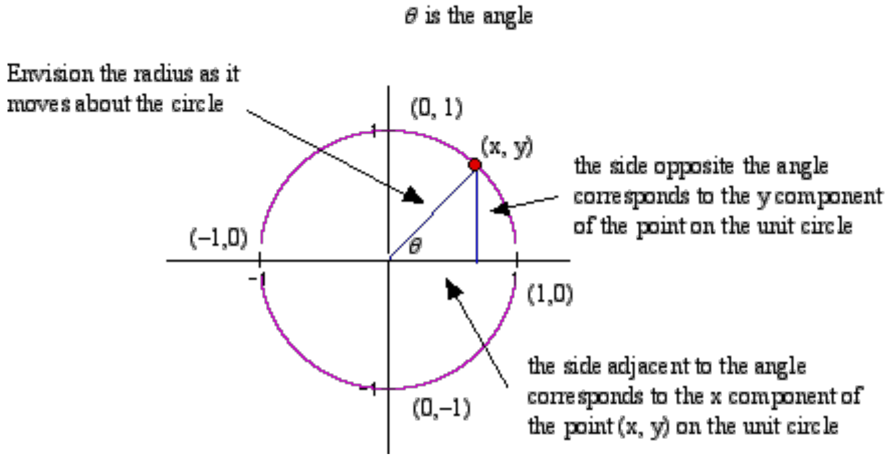
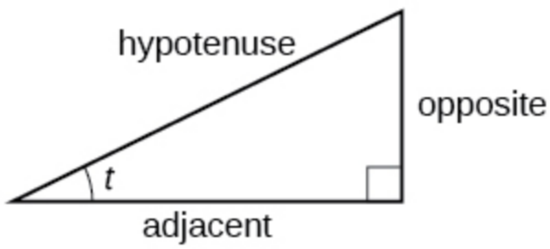
Using Right Triangles to Evaluate Trigonometric Functions

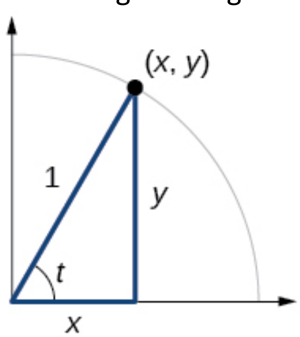


Notice that the triangle is inscribed in a circle of radius 1. Such a circle, with a center at the origin and a radius of 1, is known as a _________________ ________________________.



The adjacent side is the side closest to the angle, *x*. (Adjacent means "next to.") The opposite side is the side across from the angle, *y*. The hypotenuse is the side of the triangle opposite the right angle, 1.

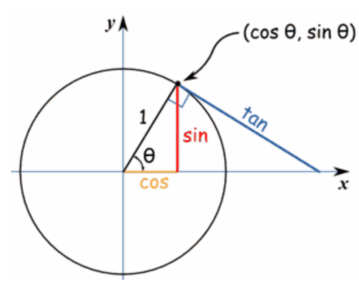
Given a right triangle with an acute angle of t, the first three trigonometric functions are…

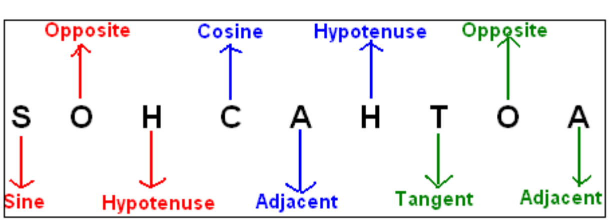


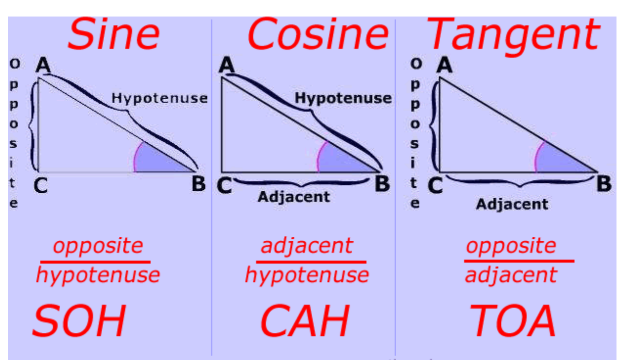
Sine $\sin t =$

Cosine $\cos t =$

Tangent $\tan t =$





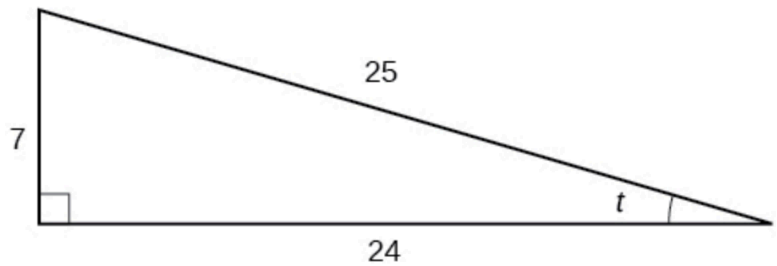


HOW TO

Given the side lengths of a right triangle and one of the acute angles, find the sine, cosine, and tangent of that angle.

1. Find the sine as the ratio of the opposite side to the hypotenuse.
2. Find the cosine as the ratio of the adjacent side to the hypotenuse.
3. Find the tangent as the ratio of the opposite side to the adjacent side.

Example: Given the triangle, find the value of $\sin t$, $\cos t$, & $\tan t$.



Reciprocal Functions

In addition to sine, cosine, and tangent, there are three more functions. These functions are the reciprocals of the first three functions.

Secant $\quad \sec t =$

Cosecant $\quad \csc t =$

Cotangent $\quad \cot t =$

$$\begin{aligned} \sin t &= \frac{1}{\csc t} & \qquad \csc t &= \frac{1}{\sin t} \\ \cos t &= \frac{1}{\sec t} & \qquad \sec t &= \frac{1}{\cos t} \\ \tan t &= \frac{1}{\cot t} & \qquad \cot t &= \frac{1}{\tan t} \end{aligned}$$

Many problems ask for all six trigonometric functions for a given angle in a triangle. A possible strategy to use is to find the sine, cosine, and tangent of the angles first. Then, find the other trigonometric functions easily using the reciprocals.

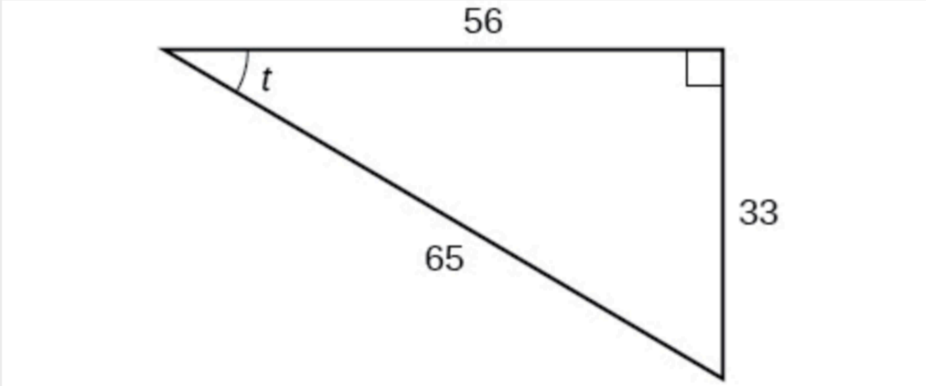
HOW TO

Given the side lengths of a right triangle, evaluate the six trigonometric functions of one of the acute angles.

1. If needed, draw the right triangle and label the angle provided.
2. Identify the angle, the adjacent side, the side opposite the angle, and the hypotenuse of the right triangle.
3. Find the required function:
    - sine as the ratio of the opposite side to the hypotenuse
    - cosine as the ratio of the adjacent side to the hypotenuse
    - tangent as the ratio of the opposite side to the adjacent side
    - secant as the ratio of the hypotenuse to the adjacent side
    - cosecant as the ratio of the hypotenuse to the opposite side
    - cotangent as the ratio of the adjacent side to the opposite side

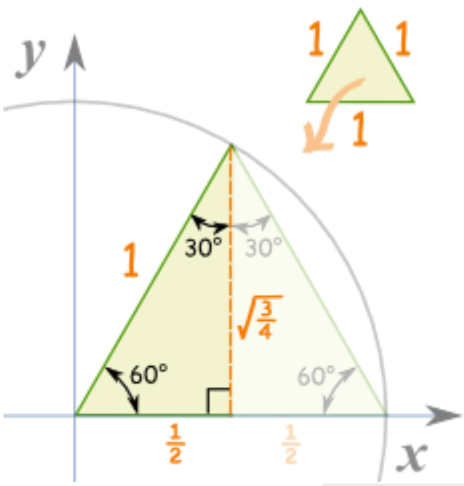
Example:

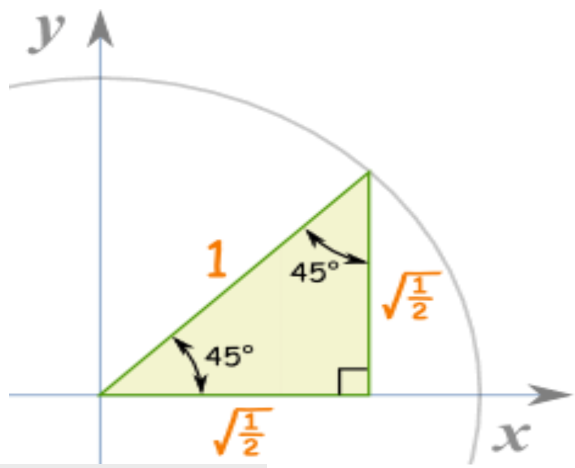
Using the triangle shown in Figure, evaluate $\sin t$, $\cos t$, $\tan t$, $\sec t$, $\csc t$, and $\cot t$.



Finding Trig Functions of Special Angles Using Side Lengths

Suppose we have a $30°, 60°, 90°$ triangle, which can also be described as a $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ triangle. sides of a $45°, 45°, 90°$ triangle, which can also be described as a $\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}$ triangle.





HOW TO

Given trigonometric functions of a special angle, evaluate using side lengths.

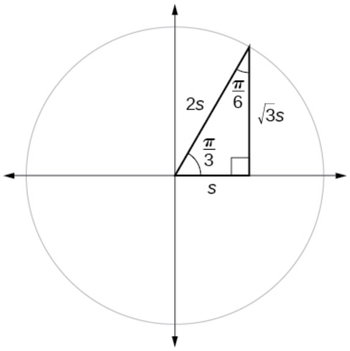
1. Use the side lengths shown in Figure for the special angle you wish to evaluate.
2. Use the ratio of side lengths appropriate to the function you wish to evaluate.

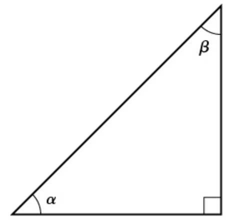
Example:

Find the exact value of the trigonometric functions of $\frac{\pi}{4}$, using side lengths.

Using Equal Cofunction Compliments

Interesting relationship. Find the *sin* and *cos* of the angles $\frac{\pi}{3}$ & $\frac{\pi}{6}$.





These relationships are known as ______________________________ _______________________________.

A GENERAL NOTE: COFUNCTION IDENTITIES

The **cofunction identities** in radians are listed in Table.

$\cos t = \sin\left(\frac{\pi}{2} - t\right)$	$\sin t = \cos\left(\frac{\pi}{2} - t\right)$
$\tan t = \cot\left(\frac{\pi}{2} - t\right)$	$\cot t = \tan\left(\frac{\pi}{2} - t\right)$
$\sec t = \csc\left(\frac{\pi}{2} - t\right)$	$\csc t = \sec\left(\frac{\pi}{2} - t\right)$

Example

If $\csc\left(\frac{\pi}{6}\right) = 2$, find $\sec\left(\frac{\pi}{3}\right)$.

Using Trigonometric Functions

HOW TO

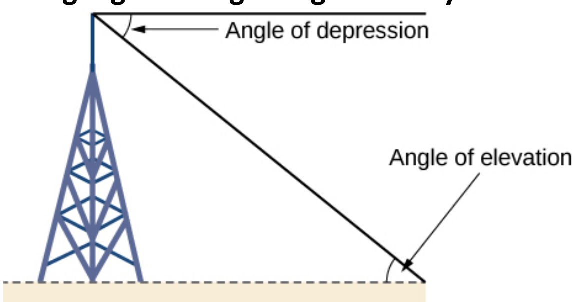
Given a right triangle, the length of one side, and the measure of one acute angle, find the remaining sides.

1. For each side, select the trigonometric function that has the unknown side as either the numerator or the denominator. The known side will in turn be the denominator or the numerator.
2. Write an equation setting the function value of the known angle equal to the ratio of the corresponding sides.
3. Using the value of the trigonometric function and the known side length, solve for the missing side length.

Example

A right triangle has one angle of $\frac{\pi}{3}$ and a hypotenuse of 20. Find the unknown sides and angle of the triangle.

Using Right Triangle Trigonometry to Solve Applied Problems



HOW TO

Given a tall object, measure its height indirectly.

1. Make a sketch of the problem situation to keep track of known and unknown information.
2. Lay out a measured distance from the base of the object to a point where the top of the object is clearly visible.
3. At the other end of the measured distance, look up to the top of the object. Measure the angle the line of sight makes with the horizontal.
4. Write an equation relating the unknown height, the measured distance, and the tangent of the angle of the line of sight.
5. Solve the equation for the unknown height.

The angle of ________________________________ of an object above an observer relative to the observer is the angle between the horizontal and the line from the object to the observer's eye. The angle of __________________________ of an object below an observer relative to the observer is the angle between the horizontal and the line from the object to the observer's eye.

These right triangles created have sides that represent the unknown height, the measured distance from the base, and the angled line of sight from the ground to the top of the object. Knowing the measured distance to the base of the object and the angle of the line of sight, we can use trigonometric functions to calculate the unknown height.

Example:

How long a ladder is needed to reach a windowsill 50 feet above the ground if the ladder rests against the building making an angle of $\frac{5\pi}{12}$ with the ground? Round to the nearest foot.