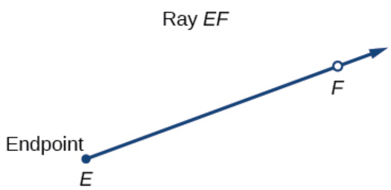
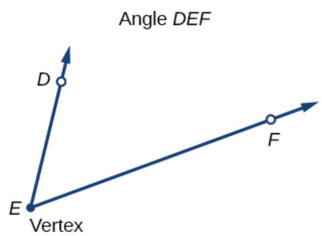
7.1 – Angles

Drawing Angles in Standard Position



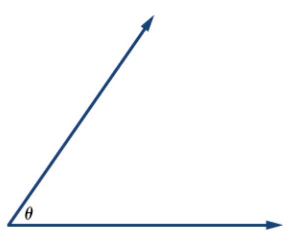
A ________________ is a directed line segment. It consists of one point on a line and all points extending in one direction from that point. The first point is called the _______________________ of the ray.

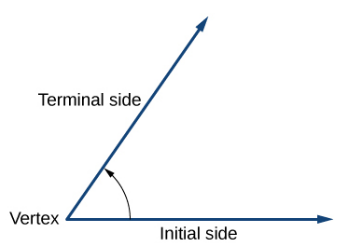


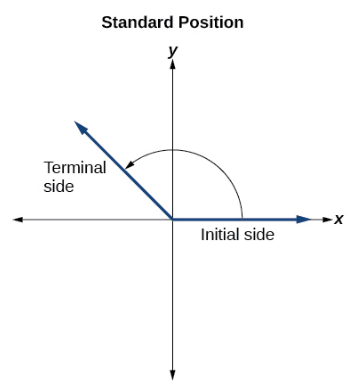
An ____________________ is the union of two rays having a common endpoint. The endpoint is called the ___________________ of the angle, and the two rays are the sides of the angle.

Greek letter are often used as variables for the measure of an angle.

θ	φ or ϕ	α	β	γ
theta	phi	alpha	beta	gamma







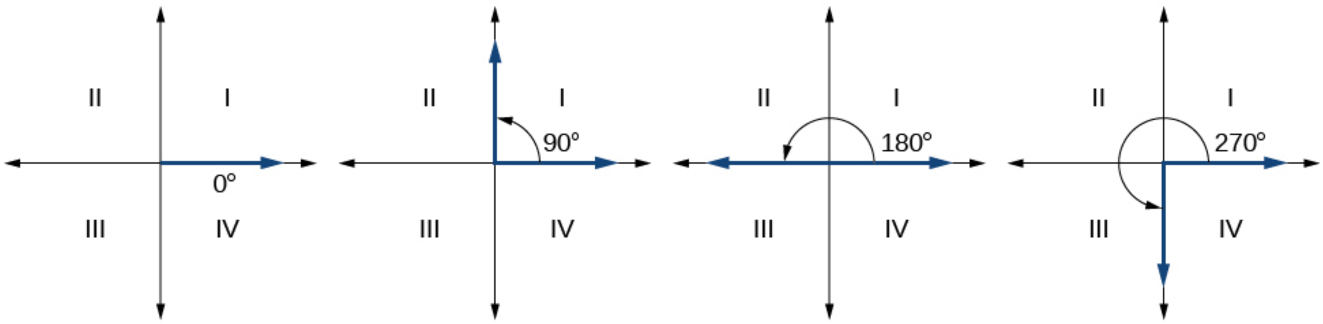
The measure of an angle is the amount of rotation from the ____________________ side to the ___________________ side. Probably the most familiar unit of angle measurement is the ________________. One degree is ___________of a circular rotation, so a complete circular rotation contains 360 degrees. An angle measured in degrees should always include the unit "degrees" after the number, or include the degree symbol °. For example, 90 degrees = 90°.

An angle is in ___________________________ position if its vertex is located at the origin, and its initial side extends along the positive *x*-axis.

If the angle is measured in a counterclockwise direction from the initial side to the terminal side, the angle is said to be a _______________________ angle. If the angle is measured in a clockwise direction, the angle is said to be a _______________________ angle.

A GENERAL NOTE: QUADRANTAL ANGLES

An angle is a **quadrantal angle** if its terminal side lies on an axis, including $0°, 90°, 180°, 270°,$ or $360°$.



Converting Between Degrees and Radians

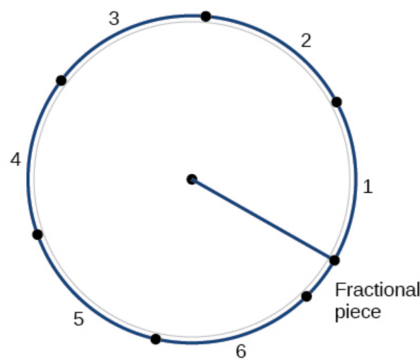
An ______________ may be a portion of a full circle, a full circle, or more than a full circle, represented by more than one full rotation. The ________________ of the arc around an entire circle is called the ____ of that circle.

The Circumference of a Circle

$$C = 2\pi r$$

$$\frac{C}{r} = 2\pi$$

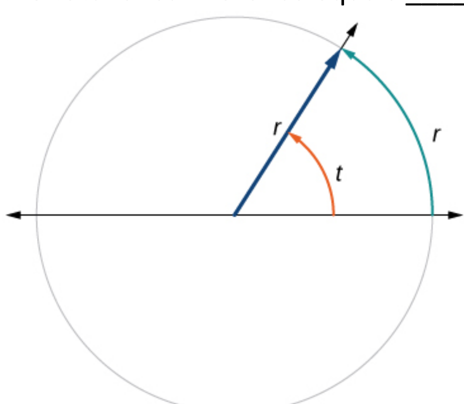
$$2\pi \approx 6.28$$



That means that if we took a string as long as the radius and used it to measure consecutive lengths around the circumference, there would be room for six full string-lengths and a little more than a quarter of a seventh.

What Is a Radian?

One _____________________________ is the measure of a central angle of a circle that intercepts an arc equal in length to the radius of that circle. A central angle is an angle formed at the center of a circle by two radii. Because the total circumference equals _______ times the ___________, a full circular rotation is 2π radians.



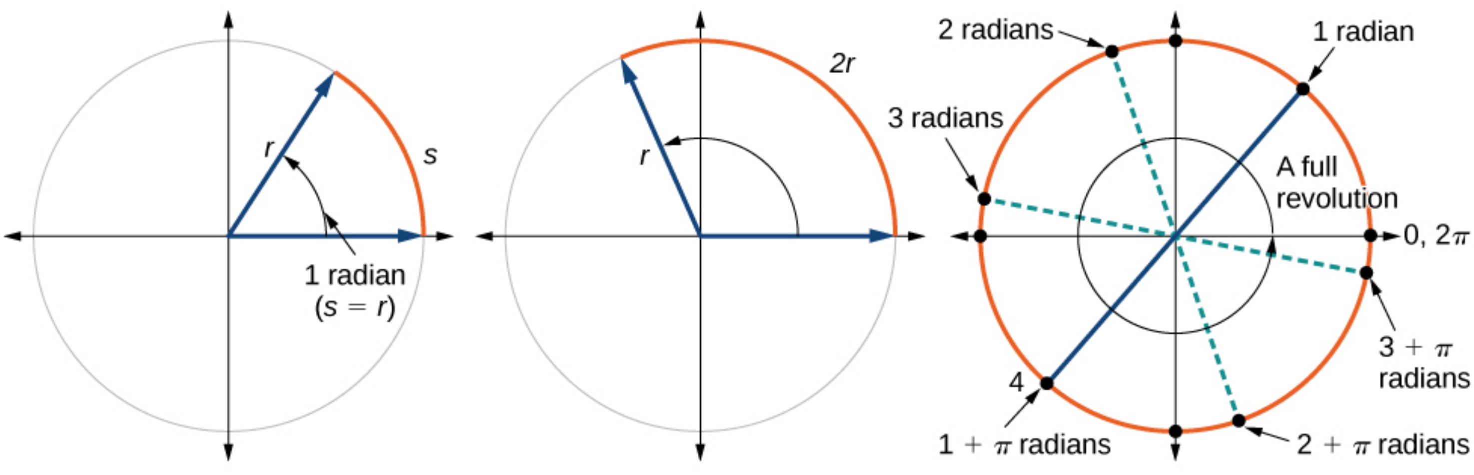
$$\begin{aligned} 1 \text{ rotation} &= 360° = 2\pi \text{ radians} \\ \tfrac{1}{2} \text{ rotation} &= 180° = \pi \text{ radians} \\ \tfrac{1}{4} \text{ rotation} &= 90° = \tfrac{\pi}{2} \text{ radians} \end{aligned}$$

Relating Arc Lengths to Radius

An ____________ ____________________, *s,* is the length of the curve along the arc of a circle.

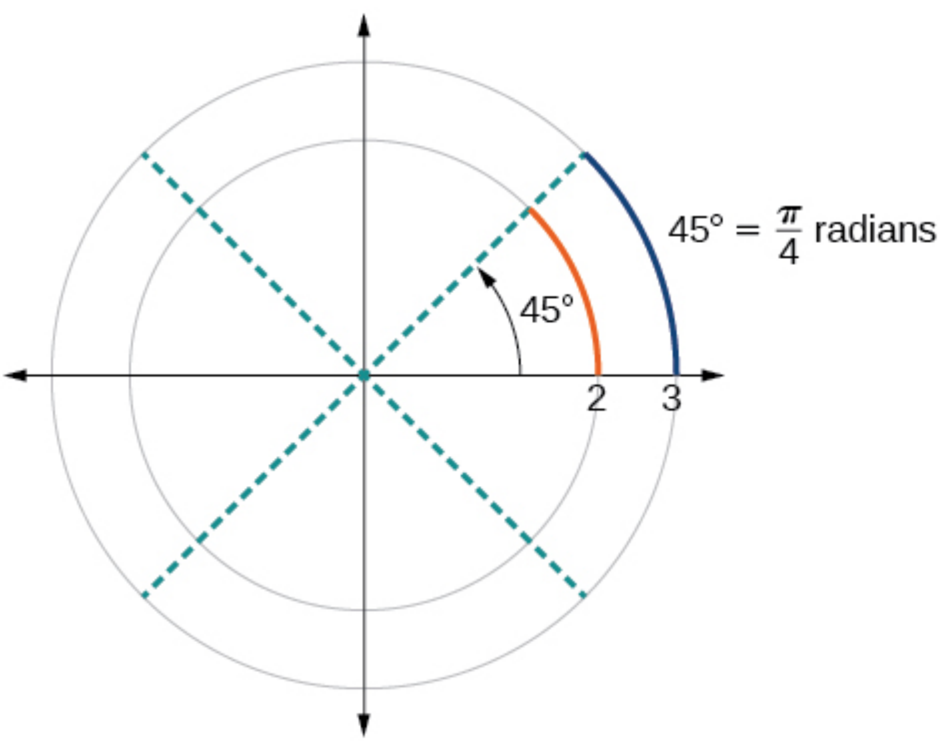
$$s = r\theta \qquad\qquad \theta = \frac{s}{r}$$

If $s = r$, then $\theta = \frac{r}{r} = 1$ radian.

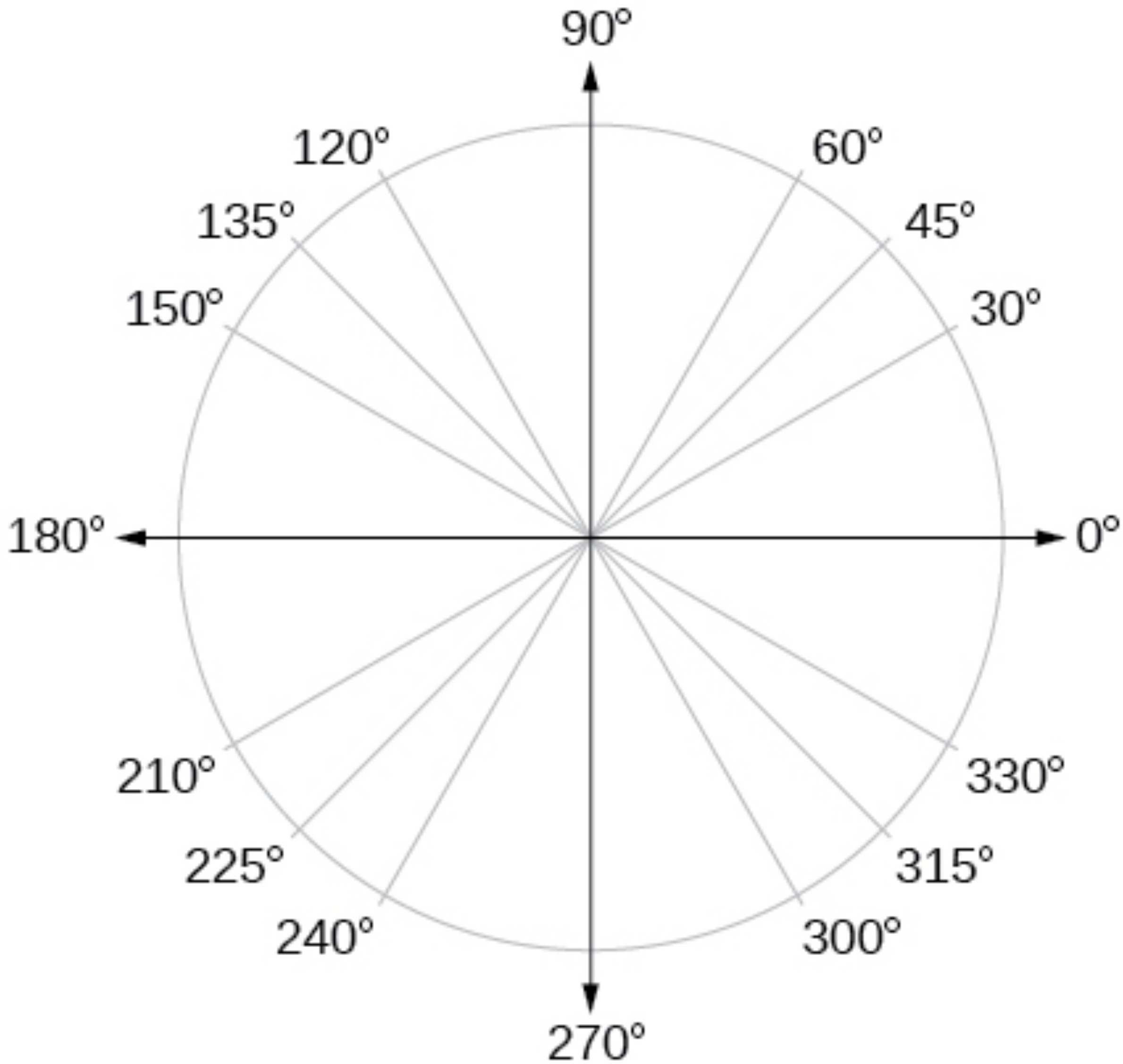


(a) (b) (c)

Ex.



Identifying Special Angles Measured in Radians



Ex.

Find the radian measure of one-third of a full rotation.

Find the radian measure of one-third of a full rotation.

Converting Between Radians and Degrees

Because degrees and radians both measure angles, we need to be able to convert between them. We can easily do so using a proportion where

θ is the measure of the angle in degrees and

θ_R is the measure of the angle in radians.

A GENERAL NOTE: CONVERTING BETWEEN RADIANS AND DEGREES

To convert between degrees and radians, use the proportion

$$\frac{\theta}{180} = \frac{\theta_R}{\pi}$$

Examples:

Convert each radian measure to degrees.

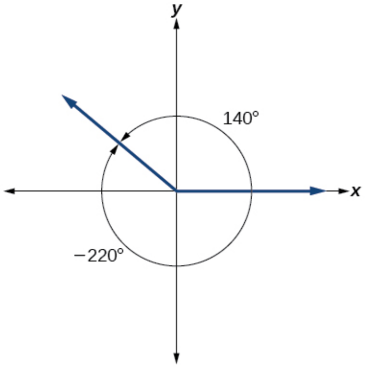
1. $\frac{\pi}{6}$
2. 3
3. $-\frac{3\pi}{4}$

Convert each degrees to radian measure.

1. 15 deg
2. 126 deg
3. -75 deg

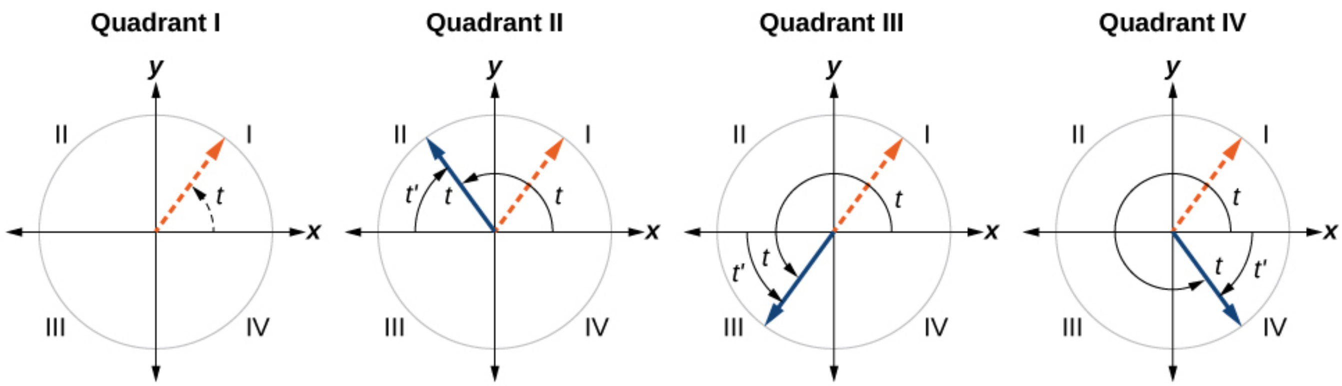
Coterminal Angles

If two angles in standard position have the same terminal side, they are __________________________ angles.



Any angle has infinitely many coterminal angles because each time we add $360°$ to that angle—or subtract $360°$ from it—the resulting value has a terminal side in the same location. For example, $100°$ and $460°$ are coterminal for this reason, as is $-260°$.

An angle's ______________________________ angle is the measure of the ___________________, ______________________, acute angle t formed by the terminal side of the angle t and the _________________________ axis.



$t' = t$

$t' = \pi - t$
$= 180° - t$

$t' = t - \pi$
$= t - 180°$

$t' = 2\pi - t$
$= 360° - t$

HOW TO

Given an angle greater than $360°$, find a coterminal angle between $0°$ and $360°$

1. Subtract $360°$ from the given angle.
2. If the result is still greater than $360°$, subtract $360°$ again till the result is between $0°$ and $360°$.
3. The resulting angle is coterminal with the original angle.

Examples

Find an angle α that is coterminal with an angle measuring $870°$, where $0° \leq \alpha < 360°$.

Find an angle β that is coterminal with an angle measuring $-300°$ such that $0° \leq \beta < 360°$.

Finding Coterminal Angles Measured in Radians

HOW TO

Given an angle greater than 2π, find a coterminal angle between 0 and 2π.

1. Subtract 2π from the given angle.
2. If the result is still greater than 2π, subtract 2π again until the result is between 0 and 2π.
3. The resulting angle is coterminal with the original angle.

Examples

Find an angle β that is coterminal with $\frac{19\pi}{4}$, where $0 \le \beta < 2\pi$.

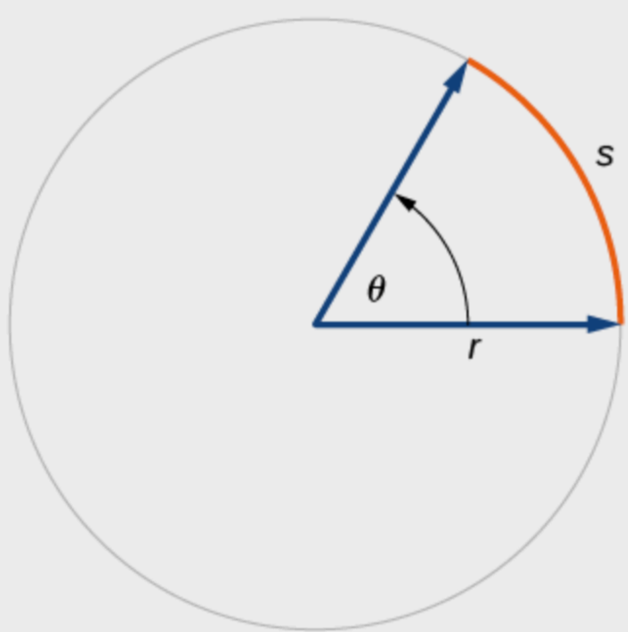
Find an angle of measure θ that is coterminal with an angle of measure $-\frac{17\pi}{6}$ where $0 \le \theta < 2\pi$.

Determining the Length of an Arc

A GENERAL NOTE: ARC LENGTH ON A CIRCLE

In a circle of radius *r*, the length of an arc s subtended by an angle with measure θ in radians, shown in Figure, is

$$s = r\theta$$



HOW TO

Given a circle of radius r, calculate the length s of the arc subtended by a given angle of measure θ.

1. If necessary, convert θ to radians.
2. Multiply the radius r θ : $s = r\theta$.

Example

Assume the orbit of Mercury around the sun is a perfect circle. Mercury is approximately 36 million miles from the sun.

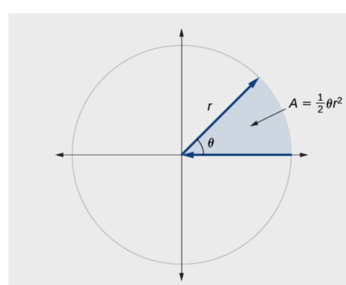
1. In one Earth day, Mercury completes 0.0114 of its total revolution. How many miles does it travel in one day?
2. Use your answer from part (a) to determine the radian measure for Mercury's movement in one Earth day.

Find the arc length along a circle of radius 10 units subtended by an angle of $215°$.

Finding the Area of a Sector

In addition to arc length, we can also use angles to find the area of a ____________________ of a circle. A sector is a region of a circle bounded by two radii and the intercepted arc, like a slice of pizza or pie.

$$\begin{aligned} \text{Area of sector} &= \left(\frac{\theta}{2\pi}\right)\pi r^2 \\ &= \frac{\theta\pi r^2}{2\pi} \\ &= \frac{1}{2}\theta r^2 \end{aligned}$$



Example

In central pivot irrigation, a large irrigation pipe on wheels rotates around a center point. A farmer has a central pivot system with a radius of 400 meters. If water restrictions only allow her to water 150 thousand square meters a day, what angle should she set the system to cover? Write the answer in radian measure to two decimal places.

Use Linear and Angular Speed to Describe Motion on a Circular Path

An object traveling in a circular path has two types of speed. **Linear speed** is speed along a straight path and can be determined by the distance it moves along (its displacement) in a given time interval.

$$v = \frac{s}{t}$$

Angular speed results from circular motion and can be determined by the angle through which a point rotates in a given time interval.

$$\omega = \frac{\theta}{t}$$

Combining the definition of angular speed with the arc length equation, $s=r\vartheta$, we can find a relationship between angular and linear speeds. The angular speed equation can be solved for ϑ, giving $\vartheta=\omega t$.

Substituting this into the arc length equation gives:

$$\begin{aligned} s &= r\theta \\ &= r\omega t \end{aligned}$$

Substituting this into the linear speed equation gives:

$$\begin{aligned} v &= \frac{s}{t} \\ &= \frac{r\omega t}{t} \\ &= r\omega \end{aligned}$$

HOW TO

Given the amount of angle rotation and the time elapsed, calculate the angular speed.

1. If necessary, convert the angle measure to radians.
2. Divide the angle in radians by the number of time units elapsed: $\omega = \frac{\theta}{t}$.
3. The resulting speed will be in radians per time unit.

Example

An old vinyl record is played on a turntable rotating clockwise at a rate of 45 rotations per minute. Find the angular speed in radians per second.

HOW TO

Given the radius of a circle, an angle of rotation, and a length of elapsed time, determine the linear speed.

1. Convert the total rotation to radians if necessary.
2. Divide the total rotation in radians by the elapsed time to find the angular speed: apply $\omega = \frac{\theta}{t}$.
3. Multiply the angular speed by the length of the radius to find the linear speed, expressed in terms of the length unit used for the radius and the time unit used for the elapsed time: apply $v = r\omega$.

Example

A satellite is rotating around Earth at 0.25 radian per hour at an altitude of 242 km above Earth. If the radius of Earth is 6378 kilometers, find the linear speed of the satellite in kilometers per hour.