5.7 – Inverses and Radical Functions

Finding the Inverse of a Polynomial Function

Two functions *f* and *g* are inverse functions if for every coordinate pair in *f*, (*a*,*b*),there exists a corresponding coordinate pair in the inverse function, *g*, (*b*,*a*).In other words, the coordinate pairs of the inverse functions have the input and output interchanged. Only one-to-one functions have inverses. Recall that a one-to-one function has a unique output value for each input value and passes the horizontal line test.

A GENERAL NOTE: VERIFYING TWO FUNCTIONS ARE INVERSES OF ONE ANOTHER

Two functions, f and g, are inverses of one another if for all x in the domain of f and g.

$$g(f(x)) = f(g(x)) = x$$

HOW TO

Given a polynomial function, find the inverse of the function by restricting the domain in such a way that the new function is one-to-one.

1. Replace $f(x)$ with y.
2. Interchange x and y.
3. Solve for y, and rename the function $f^{-1}(x)$.

Examples

Show that $f(x) = \frac{x+5}{3}$ and $f^{-1}(x) = 3x - 5$ are inverses.

Find the inverse function of $f(x) = \sqrt[3]{x+4}$.

Restricting the Domain to Find the Inverse of a Function

A GENERAL NOTE: RESTRICTING THE DOMAIN

If a function is not one-to-one, it cannot have an inverse. If we restrict the domain of the function so that it becomes one-to-one, thus creating a new function, this new function will have an inverse.

HOW TO

Given a polynomial function, restrict the domain of a function that is not one-to-one and then find the inverse.

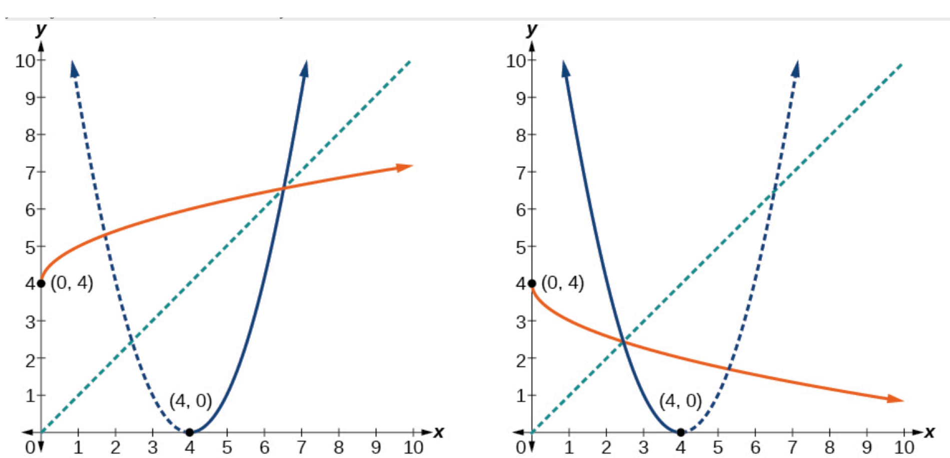
1. Restrict the domain by determining a domain on which the original function is one-to-one.
2. Replace $f(x)$ with y.
3. Interchange x and y.
4. Solve for y, and rename the function or pair of function $f^{-1}(x)$.
5. Revise the formula for $f^{-1}(x)$ by ensuring that the outputs of the inverse function correspond to the restricted domain of the original function.

Examples

Find the inverse function of f:

a. $f(x) = (x-4)^2,\ x \geq 4$

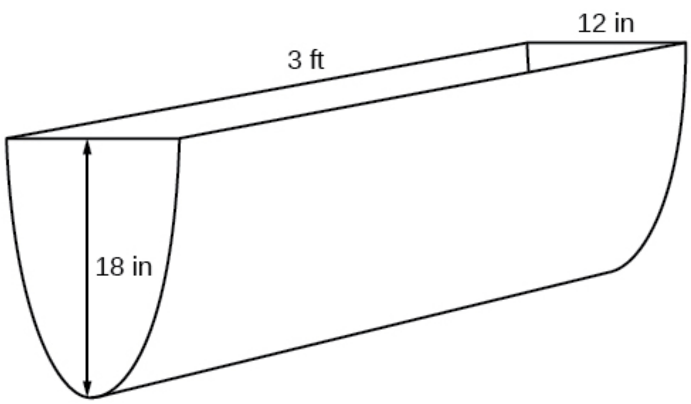
b. $f(x) = (x-4)^2,\ x \leq 4$

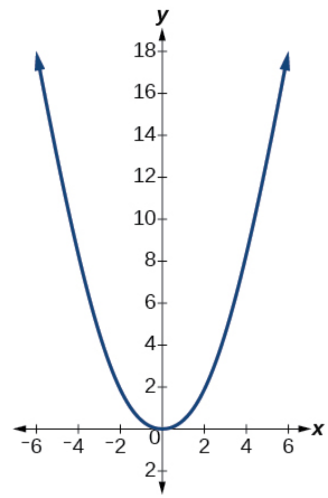


Finding the Inverse of a Polynomial Function

Two functions *f* and *g* are inverse functions if for every coordinate pair in *f*, (*a*,*b*),there exists a corresponding coordinate pair in the ______________________ function, *g*,(*b*,*a*).In other words, the coordinate pairs of the inverse functions have the input and output interchanged. Only one-to-one functions have inverses. Recall that a ______ to _______function has a unique output value for each input value and passes the horizontal line test.

For example, suppose a water runoff collector is built in the shape of a parabolic trough as shown in Figure. We can use the information in the figure to find the surface area of the water in the trough as a function of the depth of the water.





From this we find an equation for the parabolic shape. We placed the origin at the vertex of the parabola, so we know the equation will have form ______________________.Our equation will need to pass through the point (6, 18), from which we can solve for the stretch factor a.

Parabolic Cross Section Function:
Domain:
Surface Area of Water Function:

A GENERAL NOTE: VERIFYING TWO FUNCTIONS ARE INVERSES OF ONE ANOTHER

Two functions, f and g, are inverses of one another if for all x in the domain of f and g.

$$g\left(f\left(x\right)\right)=f\left(g\left(x\right)\right)=x$$

$$f^{-1}(f(x))=x, \text{for all } x \text{ in the domain of } f$$

$$f\left(f^{-1}(x)\right)=x, \text{for all } x \text{ in the domain of } f^{-1}$$

Warning:

$f^{-1}(x)$ is not the same as the reciprocal of the function $f(x)$. This use of "–1" is reserved to denote inverse functions. To denote the reciprocal of a function $f(x)$, we would need to write $(f(x))^{-1}=\frac{1}{f(x)}$.

HOW TO

Given a polynomial function, find the inverse of the function by restricting the domain in such a way that the new function is one-to-one.

1.1. Replace $f\left(x\right)$ with y.
22. Interchange x and y.
33. Solve for y, and rename the function $f^{-1}(x)$.

Show that $f\left(x\right)=\frac{1}{x+1}$ and $f^{-1}\left(x\right)=\frac{1}{x}-1$ are inverses, for $x\neq 0,-1$.

Find the inverse of the function $f(x) = 5x^3 + 1$. desmos.com

Restricting the Domain to Find the Inverse of a Polynomial Function

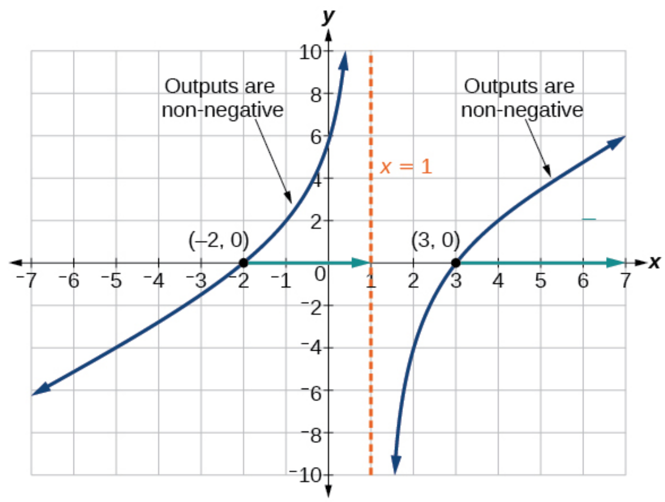
Find the inverse function of f:

1a. $f(x) = (x - 4)^2, \ x \geq 4$
2b. $f(x) = (x - 4)^2, \ x \leq 4$

Determine the Domain of a Radical Function Composed with Other Functions

Find the domain of the function $f(x) = \sqrt{\frac{(x+2)(x-3)}{(x-1)}}$.

$$\frac{(x+2)(x-3)}{(x-1)} \geq 0.$$



Solving Applications of Radical Functions

A mound of gravel is in the shape of a cone with the height equal to twice the radius. The volume of the cone in terms of the radius is given by

$$V = \frac{2}{3}\pi r^3$$

Find the inverse of the function $V = \frac{2}{3}\pi r^3$ that determines the volume V of a cone and is a function of the radius r. Then use the inverse function to calculate the radius of such a mound of gravel measuring 100 cubic feet. Use $\pi = 3.14$.

Finding the Inverse of a Rational Function

The function $C = \frac{20+0.4n}{100+n}$ represents the concentration C of an acid solution after n mL of 40% solution has been added to 100 mL of a 20% solution. First, find the inverse of the function; that is, find an expression for n in terms of C. Then use your result to determine how much of the 40% solution should be added so that the final mixture is a 35% solution.