2.6 – Other Types of Equations

Solving Equations Involving Rational Exponents

A GENERAL NOTE: RATIONAL EXPONENTS

A rational exponent indicates a power in the numerator and a root in the denominator. There are multiple ways of writing an expression, a variable, or a number with a rational exponent:

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(a^m\right)^{\frac{1}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

Examples:

a. Evaluate $64^{-\frac{1}{3}}$.

b. Solve the equation $x^{\frac{3}{2}} = 125$.

c. Solve: $(x+5)^{\frac{3}{2}} = 8$.

Solving Equations Using Factoring

A GENERAL NOTE: POLYNOMIAL EQUATIONS

A polynomial of degree *n* is an expression of the type

$$a_n x^n + a_{n-1} x^{n-1} + \cdot \cdot \cdot + a_2 x^2 + a_1 x + a_0$$

where *n* is a positive integer and $a_n, \ldots, a_0$ are real numbers and $a_n \neq 0$.

Setting the polynomial equal to zero gives a **polynomial equation**. The total number of solutions (real and complex) to a polynomial equation is equal to the highest exponent *n*.

Examples:

a. Solve by factoring: $12x^4 = 3x^2$.

b. Solve by factoring: $3x^3 - 6x^2 - 27x + 54 = 0$

Solving Radical Equations

A GENERAL NOTE: RADICAL EQUATIONS

An equation containing terms with a variable in the radicand is called a **radical equation**.

HOW TO

Given a radical equation, solve it.

1. Isolate the radical expression on one side of the equal sign. Put all remaining terms on the other side.
2. If the radical is a square root, then square both sides of the equation. If it is a cube root, then raise both sides of the equation to the third power. In other words, for an *n*th root radical, raise both sides to the *n*th power. Doing so eliminates the radical symbol.
3. Solve the remaining equation.
4. If a radical term still remains, repeat steps 1–2.
5. Confirm solutions by substituting them into the original equation.

Examples:

a. Solve the radical equation: $\sqrt{x+3} = 3x - 1$

b. Solve the equation with two radicals: $\sqrt{3x+7} + \sqrt{x+2} = 1$.

Solving Absolute Value Equations

A GENERAL NOTE: ABSOLUTE VALUE EQUATIONS

The absolute value of x is written as $|x|$. It has the following properties:

$$\text{If } x \geq 0, \text{ then } |x| = x.$$
$$\text{If } x < 0, \text{ then } |x| = -x.$$

For real numbers A and B, an equation of the form $|A| = B$, with $B \geq 0$, will have solutions when $A = B$ or $A = -B$. If $B < 0$, the equation $|A| = B$ has no solution.

An **absolute value equation** in the form $|ax + b| = c$ has the following properties:

$$\text{If } c < 0, |ax + b| = c \text{ has no solution.}$$
$$\text{If } c = 0, |ax + b| = c \text{ has one solution.}$$
$$\text{If } c > 0, |ax + b| = c \text{ has two solutions.}$$

HOW TO

Given an absolute value equation, solve it.

1. Isolate the absolute value expression on one side of the equal sign.
2. If $c > 0$, write and solve two equations: $ax + b = c$ and $ax + b = -c$.

Examples:

a. Solve the absolute value equation: $|1 - 4x| + 8 = 13$.

b. $|2x + 1| - 2 = -3$

Solving Quadratic Equations

A GENERAL NOTE: QUADRATIC FORM

If the exponent on the middle term is one-half of the exponent on the leading term, we have an **equation in quadratic form**, which we can solve as if it were a quadratic. We substitute a variable for the middle term to solve equations in quadratic form.

HOW TO

Given an equation quadratic in form, solve it.

1. Identify the exponent on the leading term and determine whether it is double the exponent on the middle term.
2. If it is, substitute a variable, such as *u*, for the variable portion of the middle term.
3. Rewrite the equation so that it takes on the standard form of a quadratic.
4. Solve using one of the usual methods for solving a quadratic.
5. Replace the substitution variable with the original term.
6. Solve the remaining equation.

a. $x^4 - 8x^2 - 9 = 0.$

b. $(x-5)^2 - 4(x-5) - 21 = 0.$

Solving Rational Equations Resulting in a Quadratic

Earlier, we solved rational equations. Sometimes, solving a rational equation results in a quadratic. When this happens, we continue the solution by simplifying the quadratic equation by one of the methods we have seen. It may turn out that there is no solution.

Solve $\frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x^2-2x}$.

Key Concepts

- Rational exponents can be rewritten several ways depending on what is most convenient for the problem. To solve, both sides of the equation are raised to a power that will render the exponent on the variable equal to 1. See Example, Example, and Example.
- Factoring extends to higher-order polynomials when it involves factoring out the GCF or factoring by grouping. See Example and Example.
- We can solve radical equations by isolating the radical and raising both sides of the equation to a power that matches the index. See Example and Example.
- To solve absolute value equations, we need to write two equations, one for the positive value and one for the negative value. See Example.
- Equations in quadratic form are easy to spot, as the exponent on the first term is double the exponent on the second term and the third term is a constant. We may also see a binomial in place of the single variable. We use substitution to solve. See Example and Example.
- Solving a rational equation may also lead to a quadratic equation or an equation in quadratic form. See Example.