2.5 – Quadratic Equations

Solving Quadratic Equations By Factoring

** This method is useful only if the quadratic expression can be ______________________. Factoring means finding expressions that can be _____________________________ together to give the expression on one side of the equation.**

When the leading coefficient is not 1, we factor a quadratic equation using the method called grouping, which requires four terms. With the equation in standard form, let's review the grouping procedures:

1. With the quadratic in standard form, $ax^2 + bx + c = 0$,multiply $a \cdot c$.
2. Find two numbers whose product equals ac and whose sum equals b.
3. Rewrite the equation replacing the bx term with two terms using the numbers found in step 1 as coefficients of *x*.
4. Factor the first two terms and then factor the last two terms. The expressions in parentheses must be exactly the same to use grouping.
5. Factor out the expression in parentheses.
6. Set the expressions equal to zero and solve for the variable.

A GENERAL NOTE: THE ZERO-PRODUCT PROPERTY AND QUADRATIC EQUATIONS

The **zero-product property** states

$$\text{If } a \cdot b = 0, \text{ then } a = 0 \text{ or } b = 0,$$

where *a* and *b* are real numbers or algebraic expressions.

A **quadratic equation** is an equation containing a second-degree polynomial; for example

$$ax^2 + bx + c = 0$$

where *a*, *b*, and *c* are real numbers, and if $a \neq 0$,it is in standard form.

Examples

Factor and solve the equation: $x^2 + x - 6 = 0$.

Solve the quadratic equation by factoring: $x^2 + 8x + 15 = 0$.

Solve by factoring: $x^2 - 25 = 0$.

Use grouping to factor and solve the quadratic equation: $4x^2 + 15x + 9 = 0$.

Solve the equation by factoring: $-3x^3 - 5x^2 - 2x = 0$.

Using the Square Root Property

A GENERAL NOTE: THE SQUARE ROOT PROPERTY

With the x^2 term isolated, the square root property states that:

$$\text{if } x^2 = k, \text{ then } x = \pm\sqrt{k}$$

where k is a nonzero real number.

HOW TO

Given a quadratic equation with an x^2 term but no x term, use the square root property to solve it.

1. Isolate the x^2 term on one side of the equal sign.
2. Take the square root of both sides of the equation, putting a $\pm$ sign before the expression on the side opposite the squared term.
3. Simplify the numbers on the side with the $\pm$ sign.

Examples

Solve the quadratic using the square root property: $x^2 = 8$.

Solve the quadratic equation: $4x^2 + 1 = 7$.

Solve the quadratic equation using the square root property: $3(x-4)^2 = 15$.

Completing the Square

Not all quadratic equations can be factored or can be solved in their original form using the square root property. In these cases, we may use a method for solving a quadratic equation known as ______________________________ the _________________. Using this method, we add or subtract terms to both sides of the equation until we have a perfect square trinomial on one side of the equal sign. We then apply the square root property. To complete the square, the leading coefficient, *a*, must equal _________. If it does not, then divide the entire equation by *a*. Then, we can use the following procedures to solve a quadratic equation by completing the square.

Solve the quadratic equation by completing the square: $x^2 - 3x - 5 = 0$.

Solve by completing the square: $x^2 - 6x = 13.$

Using the Quadratic Formula

A GENERAL NOTE: THE QUADRATIC FORMULA

Written in standard form, $ax^2 + bx + c = 0$, any quadratic equation can be solved using the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where *a*, *b*, and *c* are real numbers and $a \neq 0.$

HOW TO

Given a quadratic equation, solve it using the quadratic formula

1. Make sure the equation is in standard form: $ax^2 + bx + c = 0.$
2. Make note of the values of the coefficients and constant term, a, b, and c.
3. Carefully substitute the values noted in step 2 into the equation. To avoid needless errors, use parentheses around each number input into the formula.
4. Calculate and solve.

Use the quadratic formula to solve $x^2 + x + 2 = 0$.

Solve the quadratic equation using the quadratic formula: $9x^2 + 3x - 2 = 0$.

The Discriminant

The quadratic formula not only generates the solutions to a quadratic equation, it tells us about the nature of the solutions when we consider the ________________________________, or the expression under the radical, b^2-4ac.The discriminant tells us whether the solutions are real numbers or complex numbers, and how many solutions of each type to expect. Table relates the value of the discriminant to the solutions of a quadratic equation.

Value of Discriminant	Results
$b^2 - 4ac = 0$	One rational solution (double solution)
$b^2 - 4ac > 0$,perfect square	Two rational solutions
$b^2 - 4ac > 0$,not a perfect square	Two irrational solutions
$b^2 - 4ac < 0$	Two complex solutions

A GENERAL NOTE: THE DISCRIMINANT

For $ax^2 + bx + c = 0$,where a,b,and c are real numbers, the **discriminant** is the expression under the radical in the quadratic formula: $b^2 - 4ac$. It tells us whether the solutions are real numbers or complex numbers and how many solutions of each type to expect.

Examples

Use the discriminant to find the nature of the solutions to the following quadratic equations:

a. $x^2 + 4x + 4 = 0$
b. $8x^2 + 14x + 3 = 0$
c. $3x^2 - 5x - 2 = 0$
d. $3x^2 - 10x + 15 = 0$

Using the Pythagorean Theorem

One of the most famous formulas in mathematics is the Pythagorean Theorem. It is based on a right triangle, and states the relationship among the lengths of the sides asa^2+b2=c2, where *a* and *b* refer to the legs of a right triangle adjacent to the 90°angle, and *c* refers to the hypotenuse. It has immeasurable uses in architecture, engineering, the sciences, geometry, trigonometry, and algebra, and in everyday applications.

The Pythagorean Theorem is given as

$$a^2 + b^2 = c^2$$

where a and b refer to the legs of a right triangle adjacent to the 90° angle, and c refers to the hypotenuse, as shown in Figure.

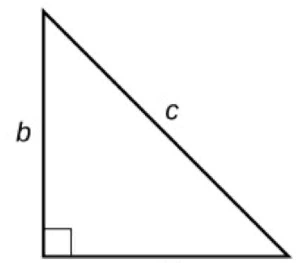

Figure 2.

Examples

Find the length of the missing side of the right triangle in Figure.

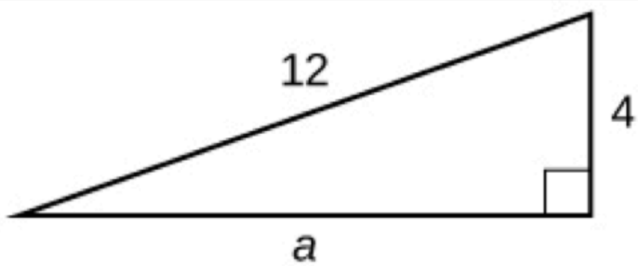

Figure 3.

Use the Pythagorean Theorem to solve the right triangle problem: Leg *a* measures 4 units, leg *b* measures 3 units. Find the length of the hypotenuse.