6.6 – Exponential and Logarithmic Equations

Using Like Bases to Solve Exponential Equations

A GENERAL NOTE: USING THE ONE-TO-ONE PROPERTY OF EXPONENTIAL FUNCTIONS TO SOLVE EXPONENTIAL EQUATIONS

For any algebraic expressions S and T, and any positive real number $b \neq 1$,

$$b^S = b^T \text{ if and only if } S = T$$

HOW TO

Given an exponential equation with the form $b^S = b^T$, where S and T are algebraic expressions with an unknown, solve for the unknown.

1.1. Use the rules of exponents to simplify, if necessary, so that the resulting equation has the form $b^S = b^T$.

22. Use the one-to-one property to set the exponents equal.

33. Solve the resulting equation, $S = T$, for the unknown.

HOW TO

Given an exponential equation with unlike bases, use the one-to-one property to solve it.

1.1. Rewrite each side in the equation as a power with a common base.

22. Use the rules of exponents to simplify, if necessary, so that the resulting equation has the form $b^S = b^T$.

33. Use the one-to-one property to set the exponents equal.

44. Solve the resulting equation, $S = T$, for the unknown.

Examples

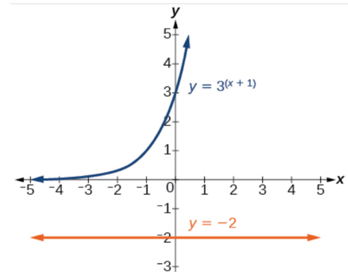
Solve $5^{2x} = 5^{3x+2}$

$3^{4x-7} = \frac{3^{2x}}{3}$

$256 = 4^{x-5}$

Solve $2^{5x} = \sqrt{2}$.

$3^{x+1} = -2$.



Solving Exponential Equations Using Logarithms

HOW TO

Given an exponential equation in which a common base cannot be found, solve for the unknown.

1. Apply the logarithm of both sides of the equation.
   - If one of the terms in the equation has base 10, use the common logarithm.
   - If none of the terms in the equation has base 10, use the natural logarithm.
2. Use the rules of logarithms to solve for the unknown.

Examples

Solve $5^{x+2} = 4^{x}$.

Solve $2^{x} = 3^{x+1}$.

$2^{x} = 3^{x}$

Equations Containing *e*

HOW TO

Given an equation of the form $y = Ae^{kt}$, solve for t.

11. Divide both sides of the equation by A.
22. Apply the natural logarithm of both sides of the equation.
33. Divide both sides of the equation by k.

Examples

Solve $100 = 20e^{2t}$.

$4e^{2x} + 5 = 12$.

$3 + e^{2t} = 7e^{2t}$.

Extraneous Solutions

Sometimes the methods used to solve an equation introduce an ________________________________solution, which is a solution that is correct algebraically but does not satisfy the conditions of the original equation. One such situation arises in solving when the logarithm is taken on both sides of the equation. In such cases, remember that the argument of the logarithm must be positive. If the number we are evaluating in a logarithm function is negative, there is no output.

Examples

$e^{2x} - e^{x} = 56$

Solve $e^{2x} = e^{x} + 2.$

Using the Definition of a Logarithm to Solve Logarithmic Equations

A GENERAL NOTE: USING THE DEFINITION OF A LOGARITHM TO SOLVE LOGARITHMIC EQUATIONS

For any algebraic expression S and real numbers b and c, where $b > 0,\ b \neq 1,$

$$\log_b(S) = c \text{ if and only if } b^c = S$$

Examples

$6 + \ln x = 10.$

$2\ln(x+1) = 10.$

Use a graphing calculator to estimate the approximate solution to the logarithmic equation $2^x = 1000$ to 2 decimal places.

Using the One-To-One Property to Solve Logarithmic Equations

A GENERAL NOTE: USING THE ONE-TO-ONE PROPERTY OF LOGARITHMS TO SOLVE LOGARITHMIC EQUATIONS

For any algebraic expressions S and T and any positive real number b, where $b \neq 1$,

$$\log_b S = \log_b T \text{ if and only if } S = T$$

Note, when solving an equation involving logarithms, always check to see if the answer is correct or if it is an extraneous solution.

HOW TO

Given an equation containing logarithms, solve it using the one-to-one property.

1.1. Use the rules of logarithms to combine like terms, if necessary, so that the resulting equation has the form $\log_b S = \log_b T$.
22. Use the one-to-one property to set the arguments equal.
33. Solve the resulting equation, $S = T$, for the unknown.

Examples

$\ln(x^2) = \ln(2x+3).$

$\ln(x^2) = \ln 1.$

Using the Formula for Radioactive Decay

$$A(t) = A_0 e^{\frac{\ln(0.5)}{T} t}$$

- A_0 is the amount initially present
- T is the half-life of the substance
- t is the time period over which the substance is studied
- y is the amount of the substance present after time t

How long will it take before twenty percent of our 1000-gram sample of uranium-235 has decayed?

uranium-235	atomic power	703,800,000 years