5.4 – Dividing Polynomials

Using Long Division to Divide Polynomials

We are familiar with the long division algorithm for ordinary arithmetic. We begin by dividing into the digits of the dividend that have the greatest place value. We divide, multiply, subtract, include the digit in the next place value position, and repeat.

$$3\overline{)178}$$

Division of polynomials that contain more than one term has similarities to long division of whole numbers. We can write a polynomial dividend as the product of the divisor and the quotient added to the remainder. The terms of the polynomial division correspond to the digits (and place values) of the whole number division. This method allows us to divide two polynomials.

$$x + 2\overline{)2x^3 - 3x^2 + 4x + 5}$$

$$\underbrace{2x^3 - 3x^2 + 4x + 5}_{\text{Dividend}} = \underbrace{(x + 2)}_{\text{Divisor}}\,\underbrace{(2x^2 - 7x + 18)}_{\text{Quotient}} + \underbrace{(-31)}_{\text{Remainder}}$$

A GENERAL NOTE: THE DIVISION ALGORITHM

The **Division Algorithm** states that, given a polynomial dividend $f(x)$ and a non-zero polynomial divisor $d(x)$ where the degree of $d(x)$ is less than or equal to the degree of $f(x)$, there exist unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x)q(x) + r(x)$$

$q(x)$ is the quotient and $r(x)$ is the remainder. The remainder is either equal to zero or has degree strictly less than $d(x)$.

If $r(x) = 0$, then $d(x)$ divides evenly into $f(x)$. This means that, in this case, both $d(x)$ and $q(x)$ are factors of $f(x)$.

HOW TO

Given a polynomial and a binomial, use long division to divide the polynomial by the binomial.

1. Set up the division problem.
2. Determine the first term of the quotient by dividing the leading term of the dividend by the leading term of the divisor.
3. Multiply the answer by the divisor and write it below the like terms of the dividend.
4. Subtract the bottom **binomial** from the top binomial.
5. Bring down the next term of the dividend.
6. Repeat steps 2–5 until reaching the last term of the dividend.
7. If the remainder is non-zero, express as a fraction using the divisor as the denominator.

Examples

Divide $5x^2 + 3x - 2$ by $x + 1$.

Divide $16x^3 - 12x^2 + 20x - 3$ by $4x + 5$.

Synthetic Division

$$
\begin{array}{r}
2x^2 - x + 18 \\
x+2\,\overline{)\,2x^3 - 3x^2 + 4x + 5} \\
\underline{-(2x^3 + 4x^2)} \\
-7x^2 + 4x \\
\underline{-(-7x^2 - 14x)} \\
18x + 5 \\
\underline{-(18x + 36)} \\
-31
\end{array}
$$

$$
\begin{array}{rrrrr}
2) & 2 & -3 & 4 & 5 \\
 & \underline{-2} & \underline{-4} & & \\
 & & \underline{-7} & \underline{14} & \\
 & & & \underline{18} & \underline{-36} \\
 & & & & -31
\end{array}
$$

Let’s look at this same example through the lens of synthetic division.

$$
\begin{array}{r|rrrr}
-2 & 2 & -3 & 4 & 5 \\
 & & & & \\
\hline
 & & & &
\end{array}
$$

A GENERAL NOTE: SYNTHETIC DIVISION

Synthetic division is a shortcut that can be used when the divisor is a binomial in the form $x - k$ where k is a real number. In **synthetic division**, only the coefficients are used in the division process.

HOW TO

Given two polynomials, use synthetic division to divide.

1. Write k for the divisor.
2. Write the coefficients of the dividend.
3. Bring the lead coefficient down.
4. Multiply the lead coefficient by k. Write the product in the next column.
5. Add the terms of the second column.
6. Multiply the result by k. Write the product in the next column.
7. Repeat steps 5 and 6 for the remaining columns.
8. Use the bottom numbers to write the quotient. The number in the last column is the remainder and has degree 0, the next number from the right has degree 1, the next number from the right has degree 2, and so on.

Examples

Use synthetic division to divide $5x^2 - 3x - 36$ by $x - 3$.

Is the first expression a factor of the second?

$x - 2,\ 3x^4 - 6x^3 - 5x + 10$

The volume of a rectangular solid is given by the polynomial $3x^4 - 3x^3 - 33x^2 + 54x$. The length of the solid is given by $3x$ and the width is given by $x - 2$. Find the height, t, of the solid.

For the following exercises, use synthetic division to determine whether the first expression is a factor of the second. If it is, indicate the factorization.

$x - \frac{1}{2}, 2x^4 - x^3 + 2x - 1$

$x + \frac{1}{3}, 3x^4 + x^3 - 3x + 1$

For the following exercises, use synthetic division to determine the quotient involving a complex number.

$\frac{x+1}{x-i}$

$\frac{x^2+1}{x-i}$