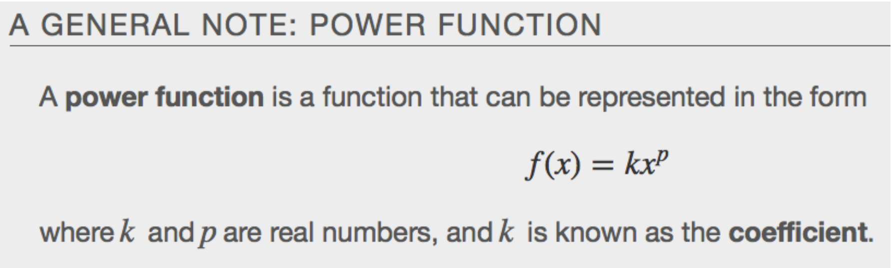
5.2 – Power Functions and Polynomial Functions

A ______________________________ **function** is a function with a _________________________ term that is the product of a real number, a **coefficient,** and a variable raised to a ________________________ real number.



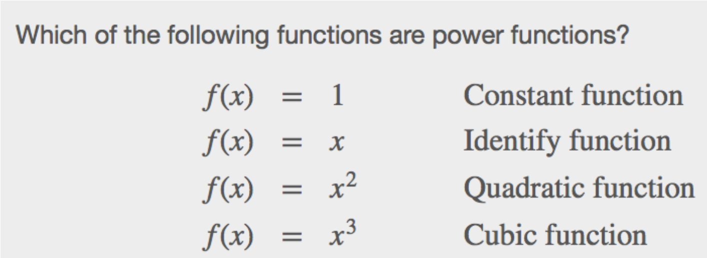
A GENERAL NOTE: POWER FUNCTION

A **power function** is a function that can be represented in the form

$$f(x) = kx^p$$

where k and p are real numbers, and k is known as the **coefficient**.

Examples



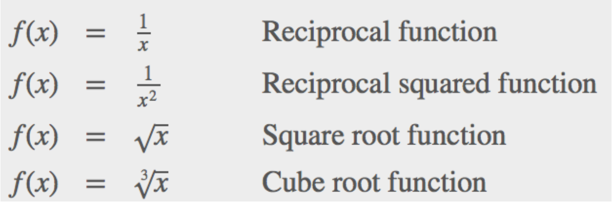
Which of the following functions are power functions?

$f(x) = 1$ Constant function

$f(x) = x$ Identify function

$f(x) = x^2$ Quadratic function

$f(x) = x^3$ Cubic function

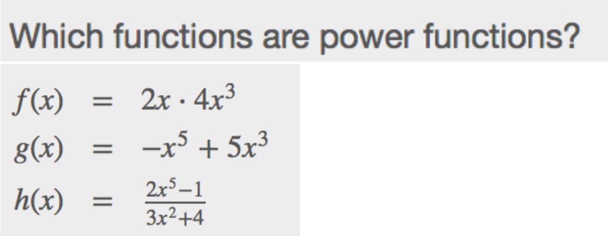


$f(x) = \frac{1}{x}$ Reciprocal function

$f(x) = \frac{1}{x^2}$ Reciprocal squared function

$f(x) = \sqrt{x}$ Square root function

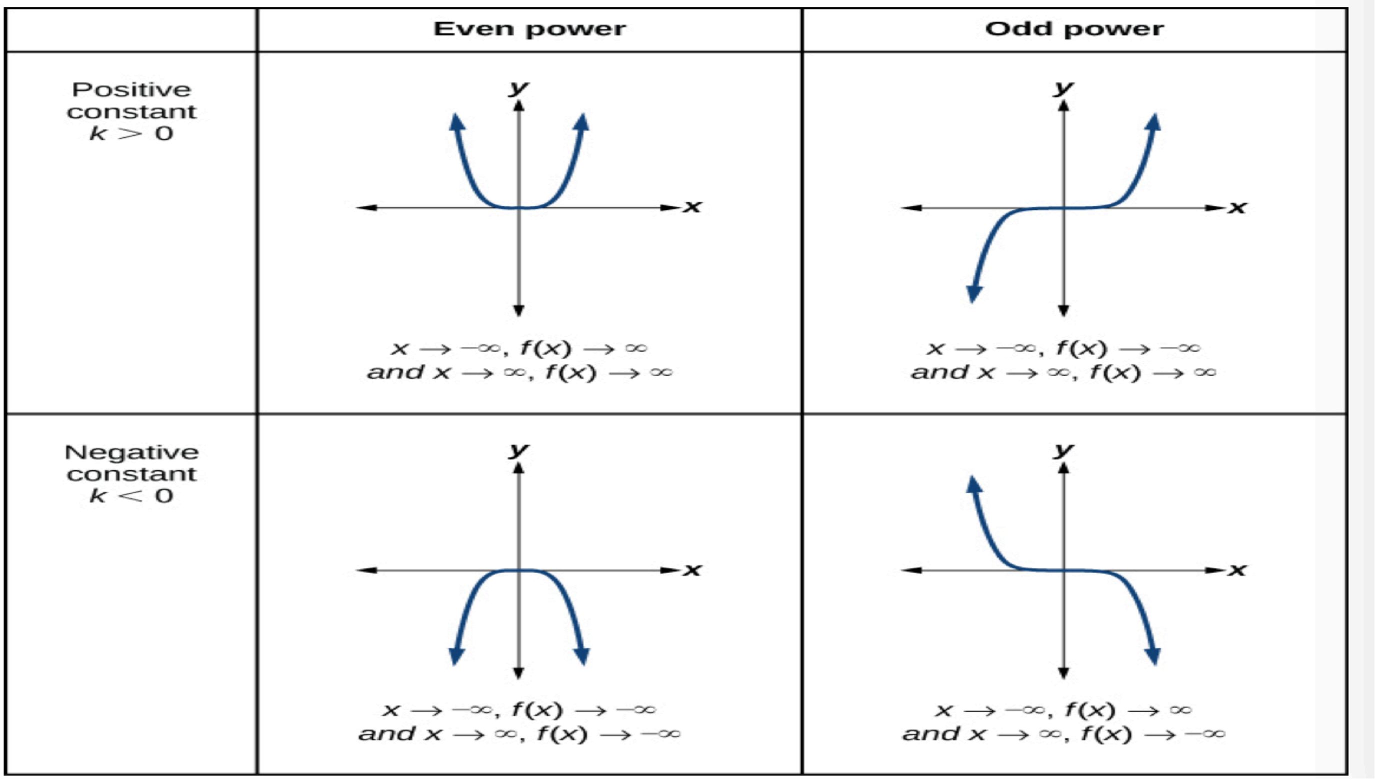
$f(x) = \sqrt[3]{x}$ Cube root function



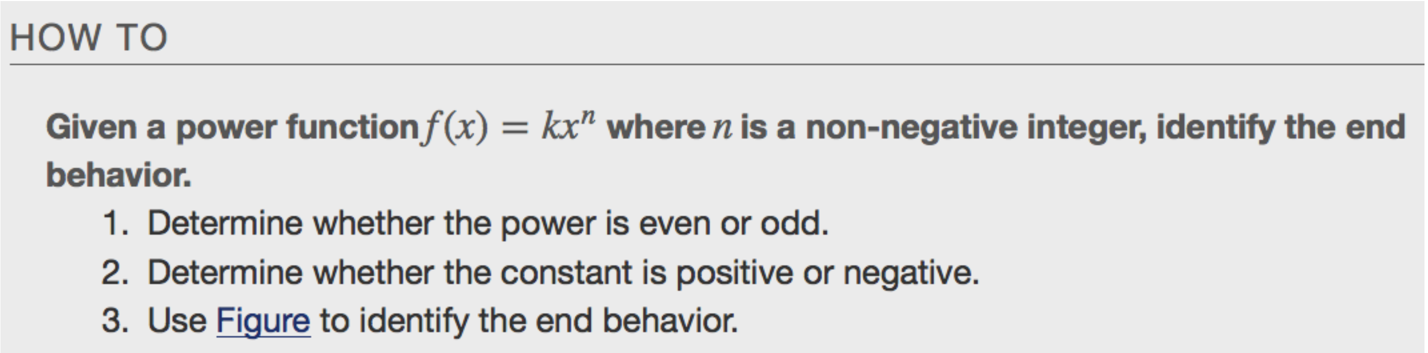
Which functions are power functions?

$$f(x) = 2x \cdot 4x^3$$
$$g(x) = -x^5 + 5x^3$$
$$h(x) = \frac{2x^5-1}{3x^2+4}$$

Identifying End Behavior of Power Functions



	Even power	Odd power
Positive constant $k > 0$	$x \to -\infty, f(x) \to \infty$ and $x \to \infty, f(x) \to \infty$	$x \to -\infty, f(x) \to -\infty$ and $x \to \infty, f(x) \to \infty$
Negative constant $k < 0$	$x \to -\infty, f(x) \to -\infty$ and $x \to \infty, f(x) \to -\infty$	$x \to -\infty, f(x) \to \infty$ and $x \to \infty, f(x) \to -\infty$

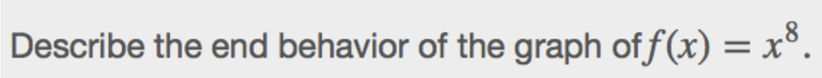


HOW TO

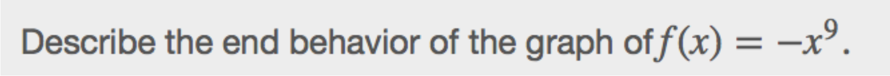
Given a power function $f(x) = kx^n$ where n is a non-negative integer, identify the end behavior.

1. Determine whether the power is even or odd.
2. Determine whether the constant is positive or negative.
3. Use Figure to identify the end behavior.

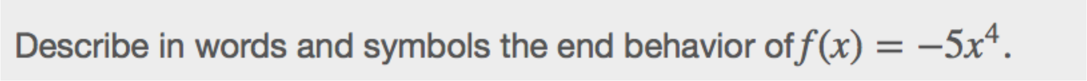
Examples



Describe the end behavior of the graph of $f(x) = x^8$.

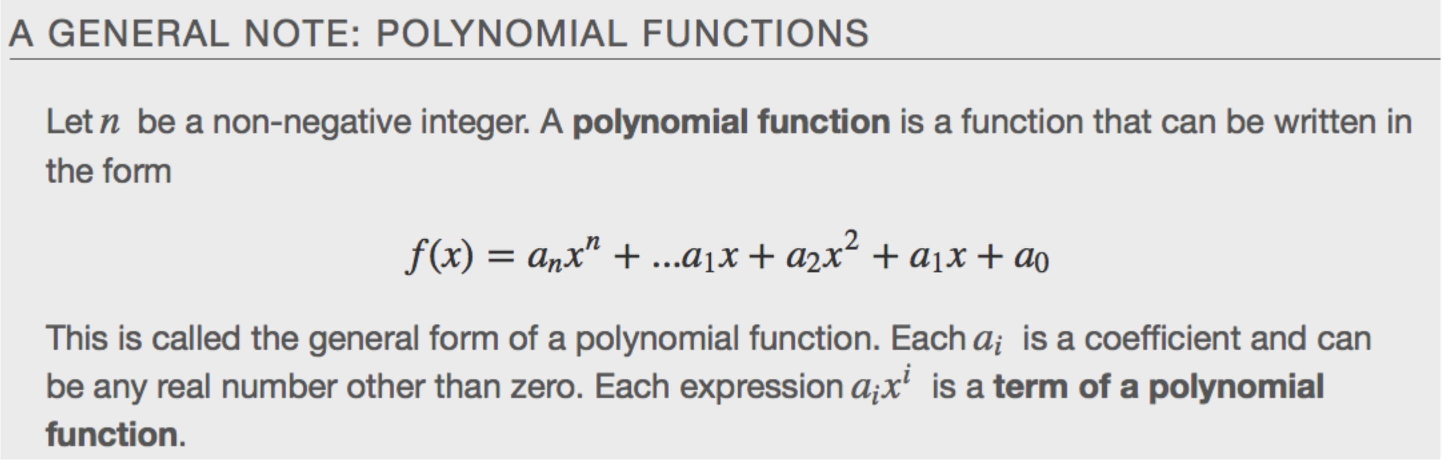


Describe the end behavior of the graph of $f(x) = -x^9$.



Describe in words and symbols the end behavior of $f(x) = -5x^4$.

Identifying Polynomial Functions



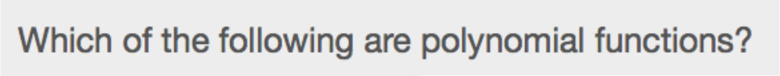
A GENERAL NOTE: POLYNOMIAL FUNCTIONS

Let n be a non-negative integer. A **polynomial function** is a function that can be written in the form

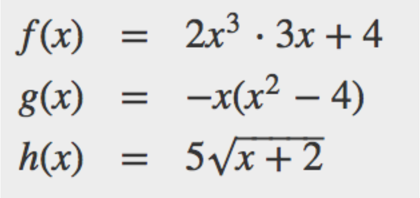
$$f(x) = a_n x^n + ...a_1 x + a_2 x^2 + a_1 x + a_0$$

This is called the general form of a polynomial function. Each a_i is a coefficient and can be any real number other than zero. Each expression $a_i x^i$ is a **term of a polynomial function**.

Example

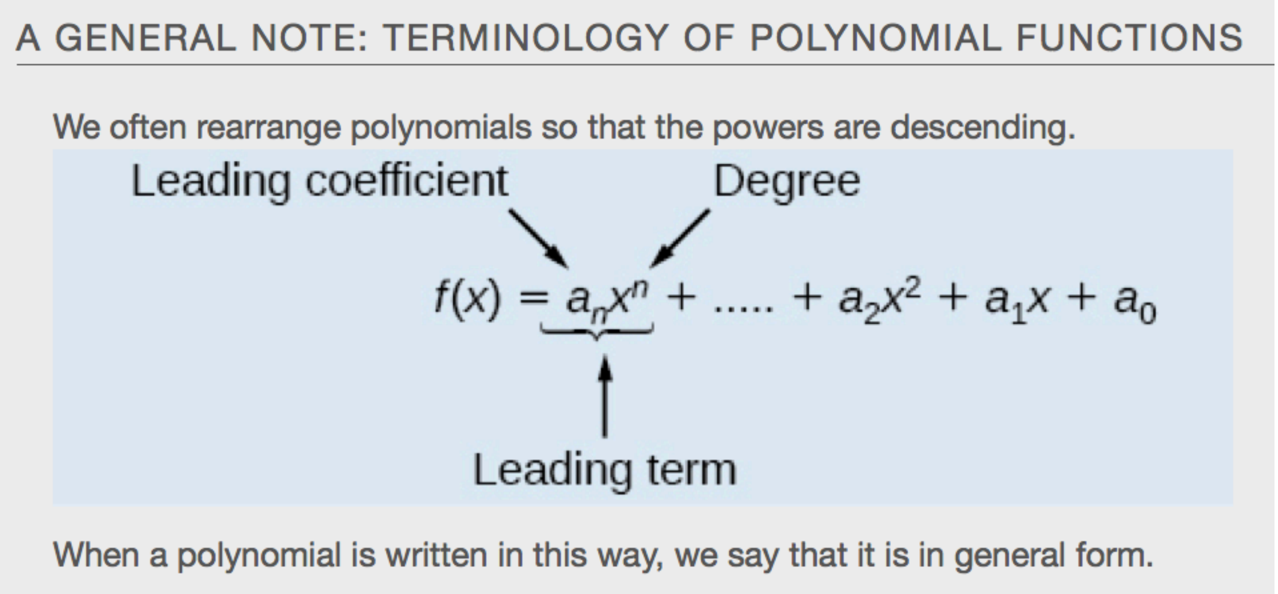


Which of the following are polynomial functions?



$$\begin{aligned} f(x) &= 2x^3 \cdot 3x + 4 \\ g(x) &= -x(x^2 - 4) \\ h(x) &= 5\sqrt{x+2} \end{aligned}$$

Identifying the Degree and Leading Coefficient of a Polynomial Function

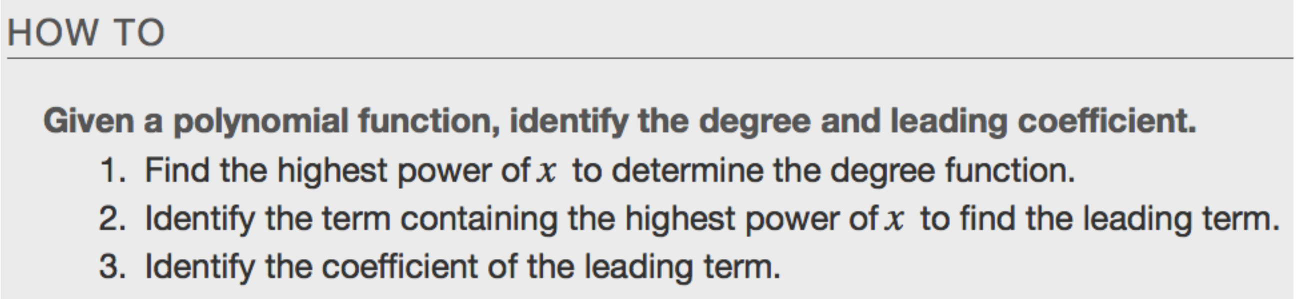


A GENERAL NOTE: TERMINOLOGY OF POLYNOMIAL FUNCTIONS

We often rearrange polynomials so that the powers are descending.

$$f(x) = \underbrace{\overset{\text{Leading coefficient}}{a_n}x^{\overset{\text{Degree}}{n}}}_{\text{Leading term}} + \ldots.. + a_2x^2 + a_1x + a_0$$

When a polynomial is written in this way, we say that it is in general form.

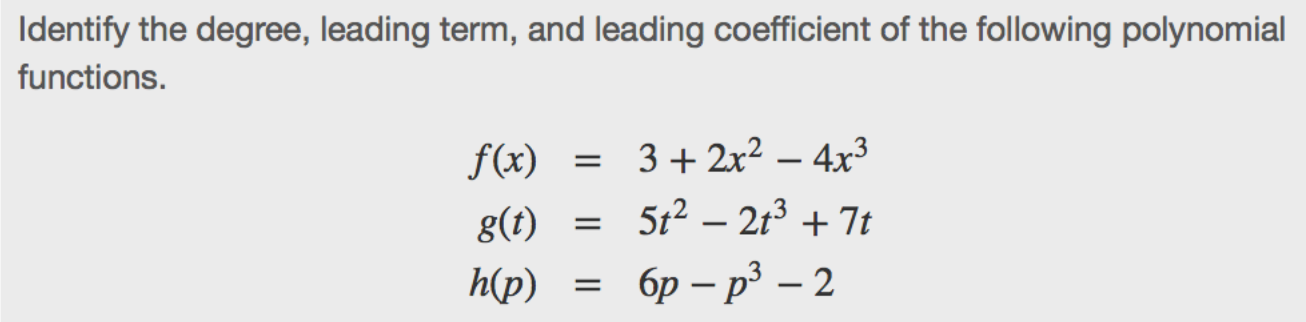


HOW TO

Given a polynomial function, identify the degree and leading coefficient.

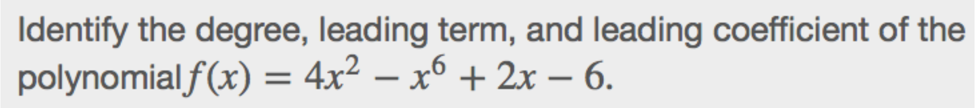
1. Find the highest power of x to determine the degree function.
2. Identify the term containing the highest power of x to find the leading term.
3. Identify the coefficient of the leading term.

Examples



Identify the degree, leading term, and leading coefficient of the following polynomial functions.

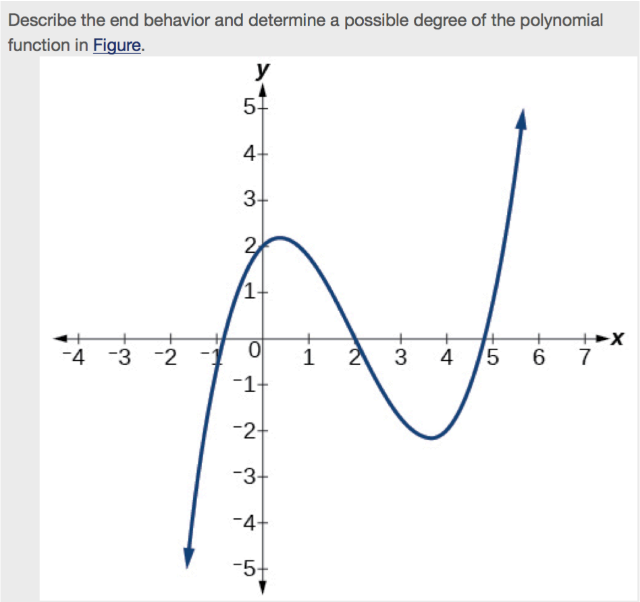
$$\begin{aligned} f(x) &= 3 + 2x^2 - 4x^3 \\ g(t) &= 5t^2 - 2t^3 + 7t \\ h(p) &= 6p - p^3 - 2 \end{aligned}$$



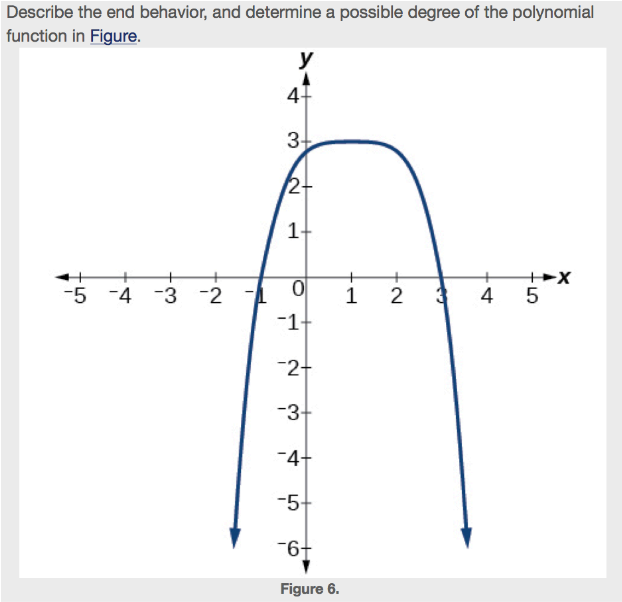
Identify the degree, leading term, and leading coefficient of the polynomial $f(x) = 4x^2 - x^6 + 2x - 6$.

Identifying End Behavior of a Polynomial Function

Knowing the degree of a polynomial function is useful in helping us predict its end behavior. To determine its end behavior, look at the ___________________________ term of the polynomial function. Because the power of the leading term is the highest, that term will grow significantly faster than the other terms as *x* gets very large or very small, so its behavior will dominate the graph. For any polynomial, the end behavior of the polynomial will match the end behavior of the __________________________ function consisting of the leading term.

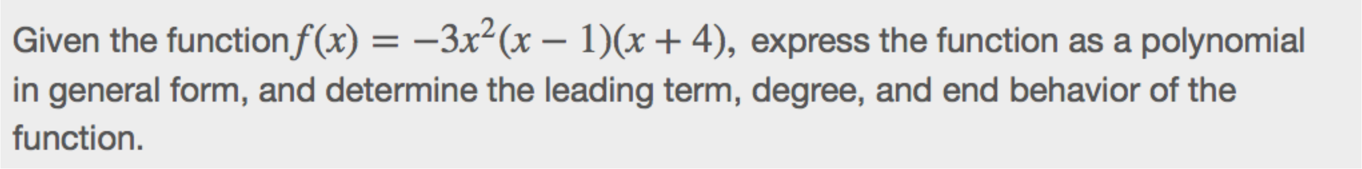


Describe the end behavior and determine a possible degree of the polynomial function in Figure.

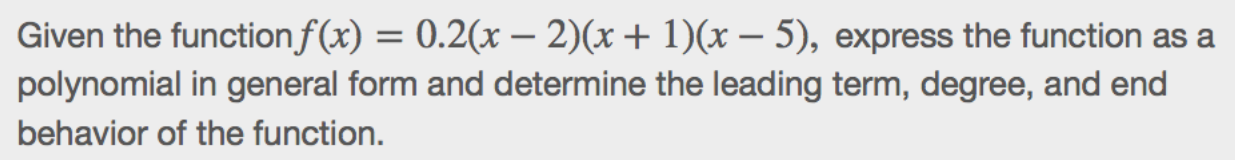


Describe the end behavior, and determine a possible degree of the polynomial function in Figure.

Figure 6.

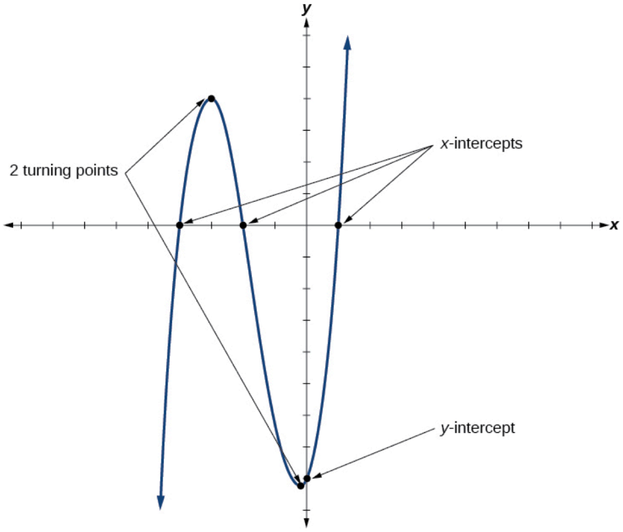


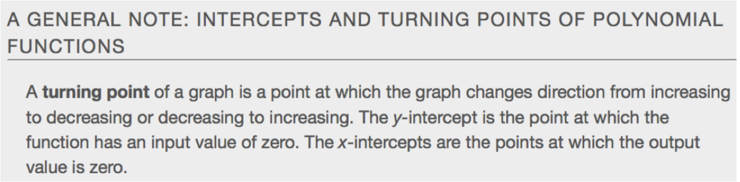
Given the function $f(x) = -3x^2(x-1)(x+4)$, express the function as a polynomial in general form, and determine the leading term, degree, and end behavior of the function.



Given the function $f(x) = 0.2(x-2)(x+1)(x-5)$, express the function as a polynomial in general form and determine the leading term, degree, and end behavior of the function.

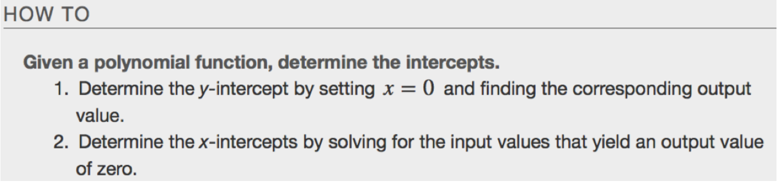
Identifying Local Behavior of Polynomial Functions





A GENERAL NOTE: INTERCEPTS AND TURNING POINTS OF POLYNOMIAL FUNCTIONS

A **turning point** of a graph is a point at which the graph changes direction from increasing to decreasing or decreasing to increasing. The *y*-intercept is the point at which the function has an input value of zero. The *x*-intercepts are the points at which the output value is zero.

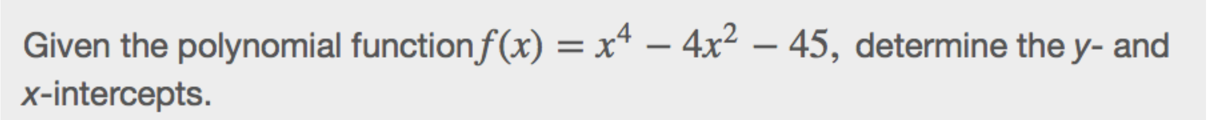


HOW TO

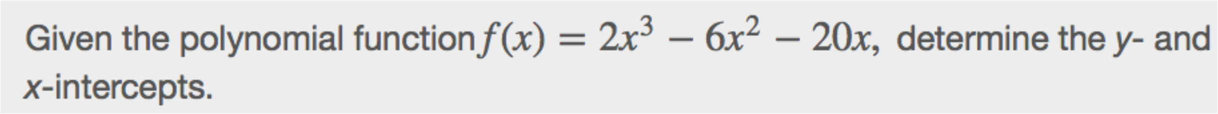
Given a polynomial function, determine the intercepts.

1. Determine the *y*-intercept by setting $x = 0$ and finding the corresponding output value.
2. Determine the *x*-intercepts by solving for the input values that yield an output value of zero.

Examples



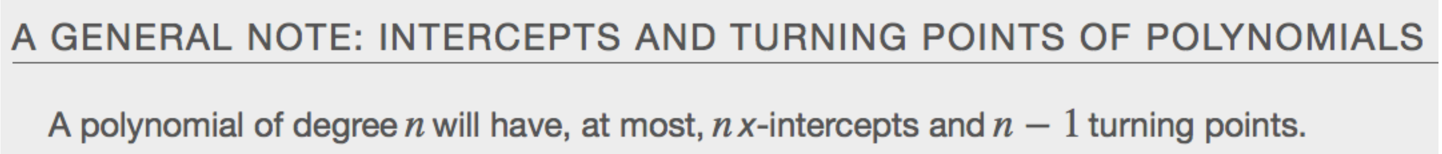
Given the polynomial function $f(x) = x^4 - 4x^2 - 45,$ determine the *y*- and *x*-intercepts.



Given the polynomial function $f(x) = 2x^3 - 6x^2 - 20x,$ determine the *y*- and *x*-intercepts.

Comparing Smooth and Continuous Graphs

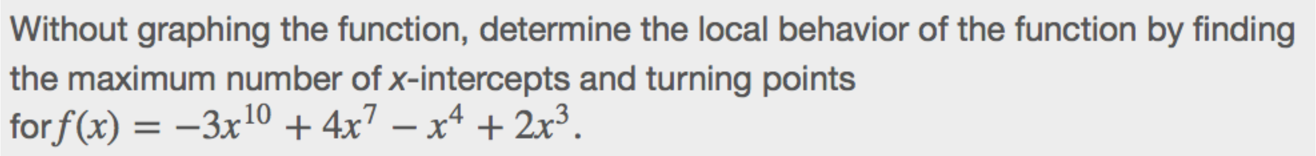
A ____ function has no breaks in its graph: the graph can be drawn without lifting the pen from the paper. A smooth curve is a graph that has no sharp ________________________. The turning points of a smooth graph must always occur at rounded curves. The graphs of polynomial functions are both continuous and smooth.



A GENERAL NOTE: INTERCEPTS AND TURNING POINTS OF POLYNOMIALS

A polynomial of degree n will have, at most, n *x*-intercepts and $n - 1$ turning points.

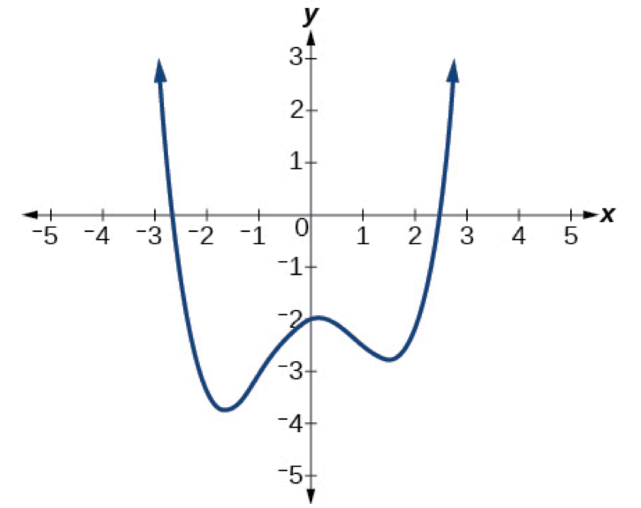
Examples



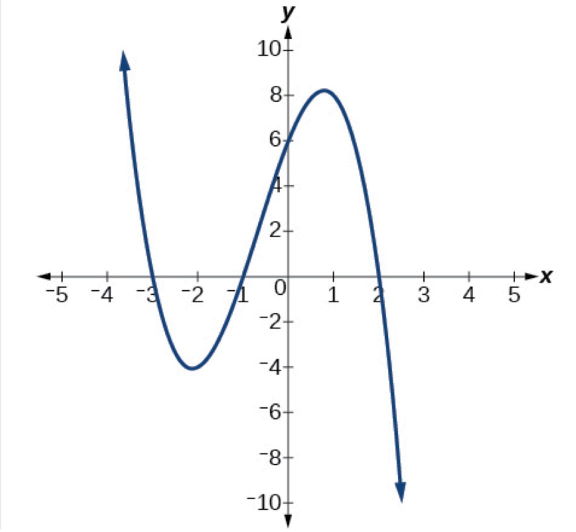
Without graphing the function, determine the local behavior of the function by finding the maximum number of *x*-intercepts and turning points for $f(x) = -3x^{10} + 4x^7 - x^4 + 2x^3.$

Without graphing the function, determine the maximum number of *x*-intercepts and turning points for $f(x) = 108 - 13x^9 - 8x^4 + 14x^{12} + 2x^3$.

What can we conclude about the polynomial represented by the graph shown in Figure based on its intercepts and turning points?



What can we conclude about the polynomial represented by the graph shown in Figure based on its intercepts and turning points?



Given the function $f(x) = -4x\,(x+3)\,(x-4)$, determine the local behavior.

Given the function $f(x) = 0.2(x-2)(x+1)(x-5)$, determine the local behavior.

For the following exercises, use the information about the graph of a polynomial function to determine the function. Assume the leading coefficient is 1 or –1. There may be more than one correct answer.

The y-intercept is $(0, 0)$. The x-intercepts are $(0, 0)$, $(2, 0)$. Degree is 3.

End behavior: as $x \to -\infty$, $f(x) \to -\infty$, as $x \to \infty$, $f(x) \to \infty$.