

5.1 – Quadratic Functions

Recognizing Characteristics of Parabolas

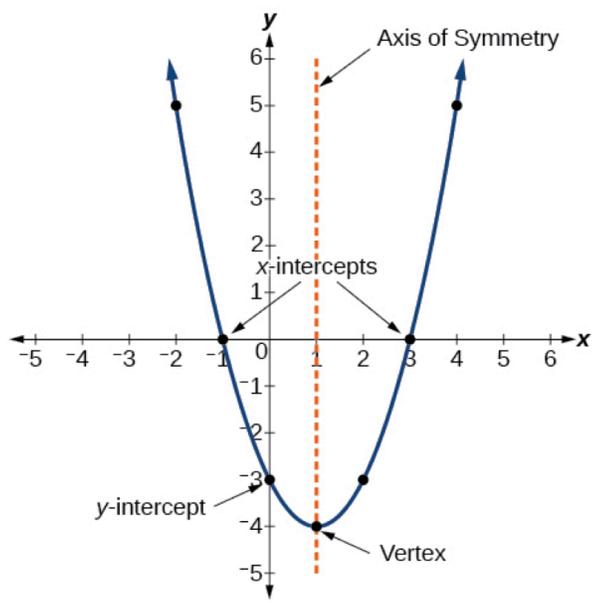


Figure 2.

A quadratic function is a polynomial function of degree two. The graph of a **quadratic function** is a parabola.

The **standard form of a quadratic function** is $f(x) = a(x - h)^2 + k$ where $a \neq 0$.

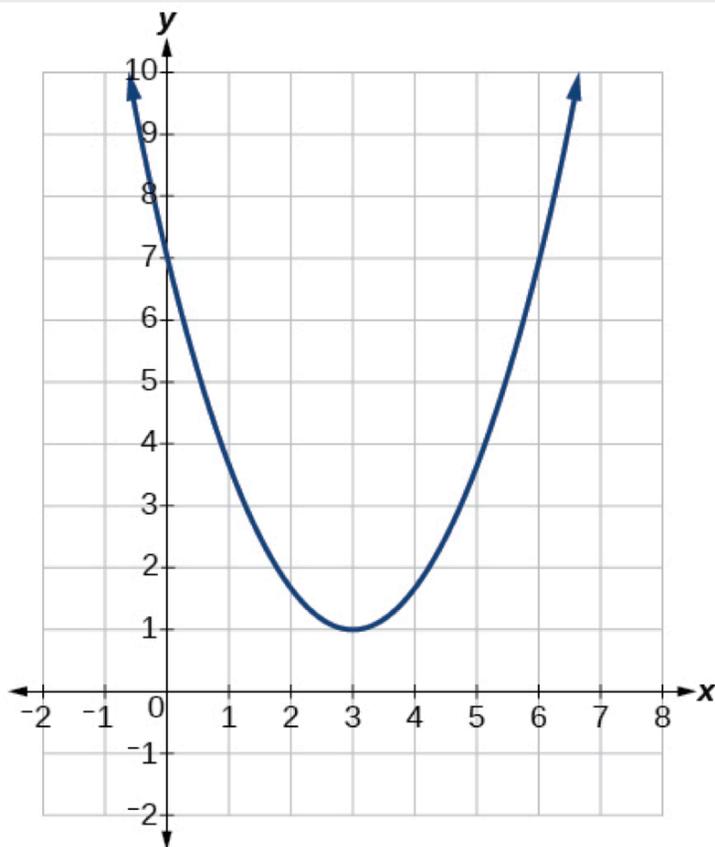
Vertex:

The **general form of a quadratic function** is $f(x) = ax^2 + bx + c$ where a , b , and c are real numbers and $a \neq 0$.

Vertex:

Example

Determine the vertex, axis of symmetry, zeros, and y-intercept of the parabola shown in Figure.



HOW TO

Given a graph of a quadratic function, write the equation of the function in general form.

1. Identify the horizontal shift of the parabola; this value is h . Identify the vertical shift of the parabola; this value is k .
2. Substitute the values of the horizontal and vertical shift for h and k . in the function $f(x) = a(x-h)^2 + k$.
3. Substitute the values of any point, other than the vertex, on the graph of the parabola for x and $f(x)$.
4. Solve for the stretch factor, $|a|$.
5. Expand and simplify to write in general form.

Examples

Write an equation for the quadratic function g in [Figure](#) as a transformation of $f(x) = x^2$, and then expand the formula, and simplify terms to write the equation in general form.

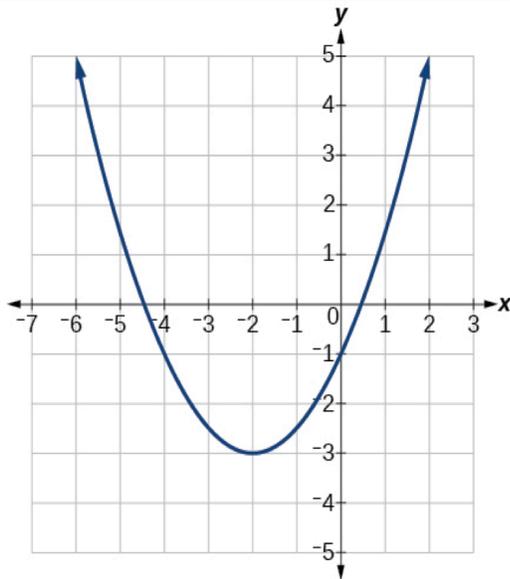
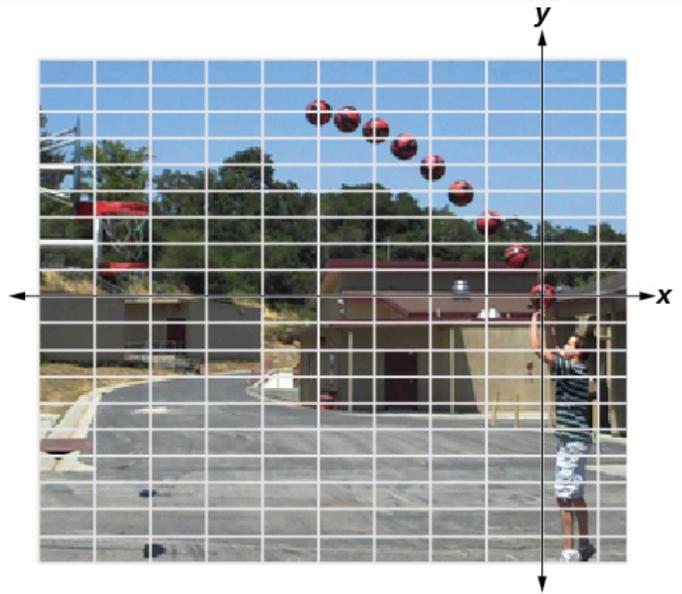


Figure 7.

| | | | | | |
|-----|----|----|----|----|---|
| x | -6 | -4 | -2 | 0 | 2 |
| y | 5 | -1 | -3 | -1 | 5 |

A coordinate grid has been superimposed over the quadratic path of a basketball in [Figure](#). Find an equation for the path of the ball. Does the shooter make the basket?



Finding the Vertex (Maximum/Minimum) of a Parabola

HOW TO

Given a quadratic function in general form, find the vertex of the parabola.

1. Identify a , b , and c .
2. Find h , the x -coordinate of the vertex, by substituting a and b into $h = -\frac{b}{2a}$.
3. Find k , the y -coordinate of the vertex, by evaluating $k = f(h) = f\left(-\frac{b}{2a}\right)$.

Examples

Find the vertex of the quadratic function $f(x) = 2x^2 - 6x + 7$. Rewrite the quadratic in standard form (vertex form).

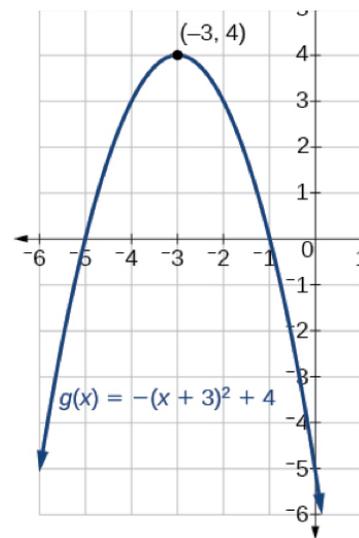
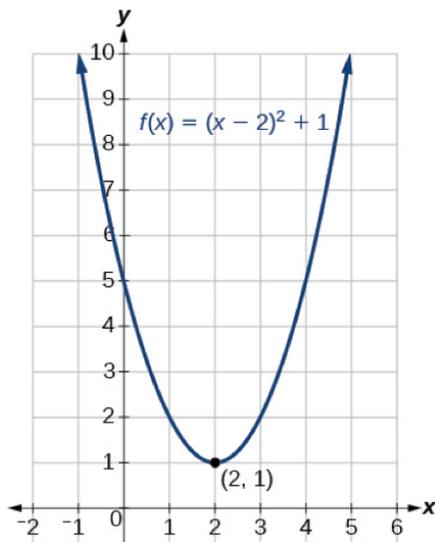
Given the equation $g(x) = 13 + x^2 - 6x$, write the equation in general form and then in standard form.

Identifying Domain and Range of a Quadratic Function

HOW TO

Given a quadratic function, find the domain and range.

1. Identify the domain of any quadratic function as all real numbers.
2. Determine whether a is positive or negative. If a is positive, the parabola has a minimum. If a is negative, the parabola has a maximum.
3. Determine the maximum or minimum value of the parabola, k .
4. If the parabola has a minimum, the range is given by $f(x) \geq k$, or $[k, \infty)$. If the parabola has a maximum, the range is given by $f(x) \leq k$, or $(-\infty, k]$.



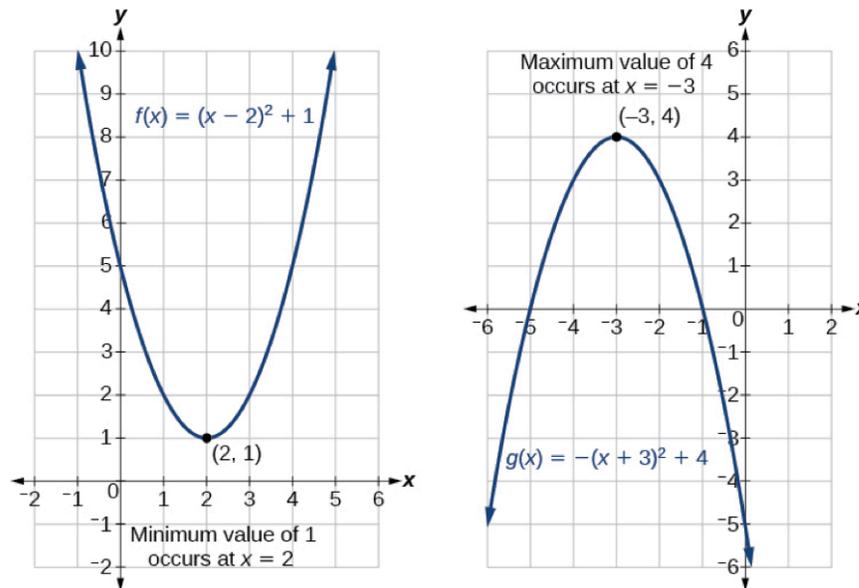
Examples

Find the domain and range of $f(x) = -5x^2 + 9x - 1$.

Find the domain and range of $f(x) = 2\left(x - \frac{4}{7}\right)^2 + \frac{8}{11}$.

Determining the Maximum and Minimum Values of Quadratic Functions

The output of the quadratic function at the vertex is the maximum or minimum value of the function, depending on the orientation of the parabola. We can see the maximum and minimum values in [Figure](#).



Finding the Maximum Value of a Quadratic Function

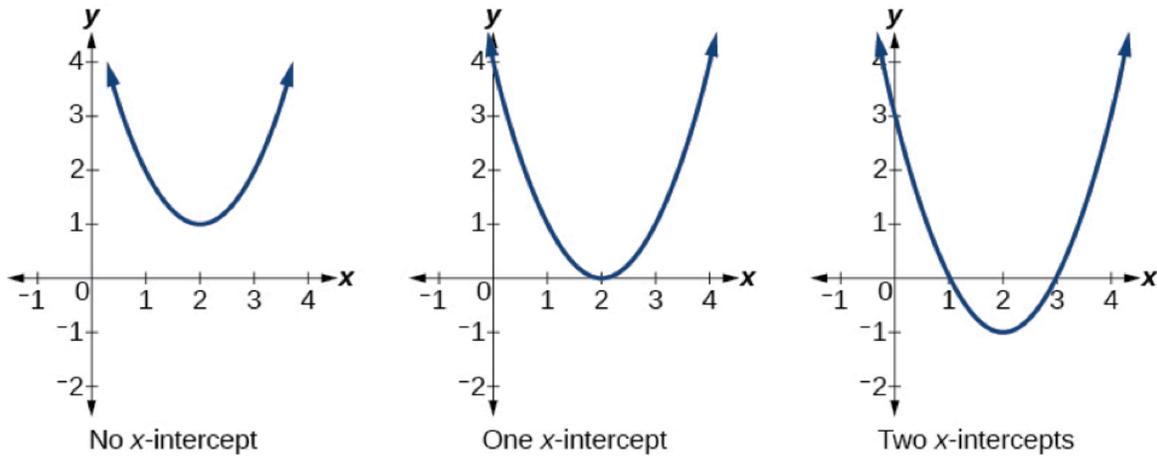
A backyard farmer wants to enclose a rectangular space for a new garden within her fenced backyard. She has purchased 80 feet of wire fencing to enclose three sides, and she will use a section of the backyard fence as the fourth side.

- Find a formula for the area enclosed by the fence if the sides of fencing perpendicular to the existing fence have length L .
- What dimensions should she make her garden to maximize the enclosed area?

Finding the x- and y-Intercepts of a Quadratic Function

Much as we did in the application problems above, we also need to find intercepts of quadratic equations for graphing parabolas. Recall that we find the y-intercept of a quadratic by evaluating the function at an input of _____, and we find the x-intercepts at locations where the _____ is zero.

Notice in [Figure](#) that the number of x-intercepts can vary depending upon the location of the graph.



Example

Find the y- and x-intercepts of the quadratic $f(x) = 3x^2 + 5x - 2$.

In a [Try It](#), we found the standard and general form for the function $g(x) = 13 + x^2 - 6x$. Now find the y - and x -intercepts (if any).

A rock is thrown upward from the top of a 112-foot high cliff overlooking the ocean at a speed of 96 feet per second. The rock's height above ocean can be modeled by the equation $H(t) = -16t^2 + 96t + 112$.

- a. When does the rock reach the maximum height?
- b. What is the maximum height of the rock?
- c. When does the rock hit the ocean?