**3.5 – Transformations of Functions**

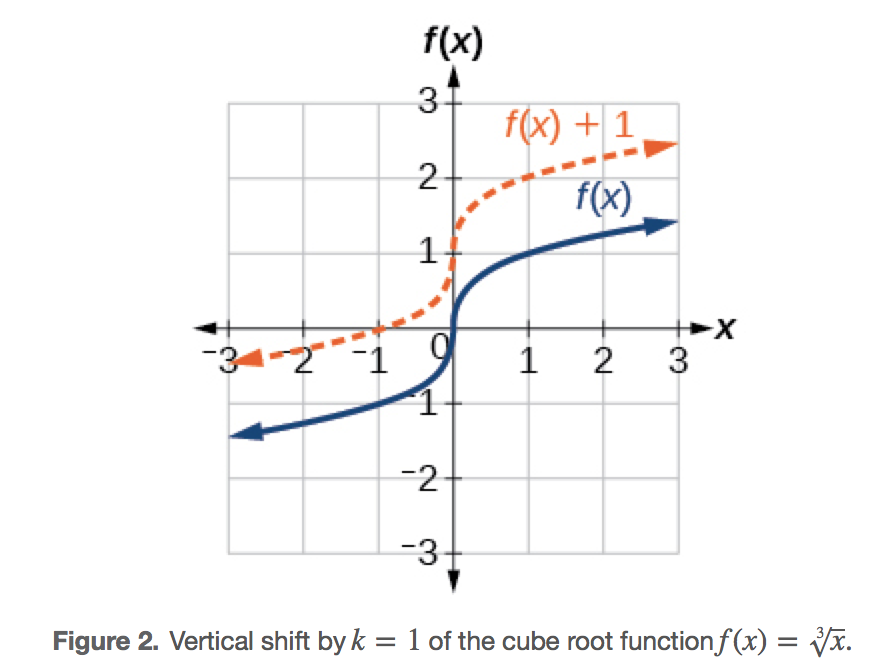
Like a carnival funhouse mirror, it presents us with a distorted image of ourselves, stretched or compressed horizontally or vertically. In a similar way, we can distort or transform mathematical functions to better adapt them to describing objects or processes in the real world. In this section, we will take a look at several kinds of transformations.

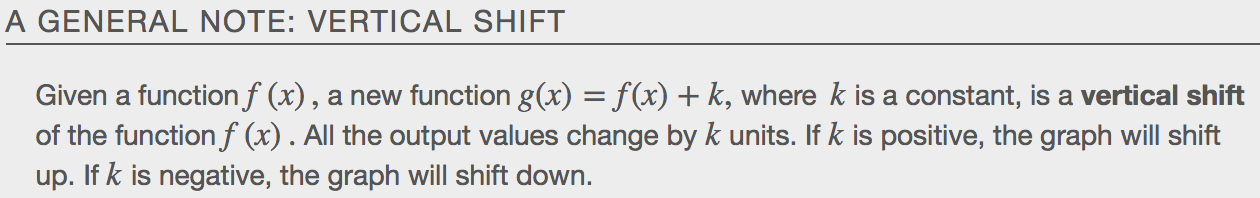
**Graphing Functions Using Vertical and Horizontal Shifts**

One method we can employ is to adapt the basic graphs of the toolkit functions to build new models for a given scenario. There are systematic ways to alter functions to construct appropriate models for the problems we are trying to solve.

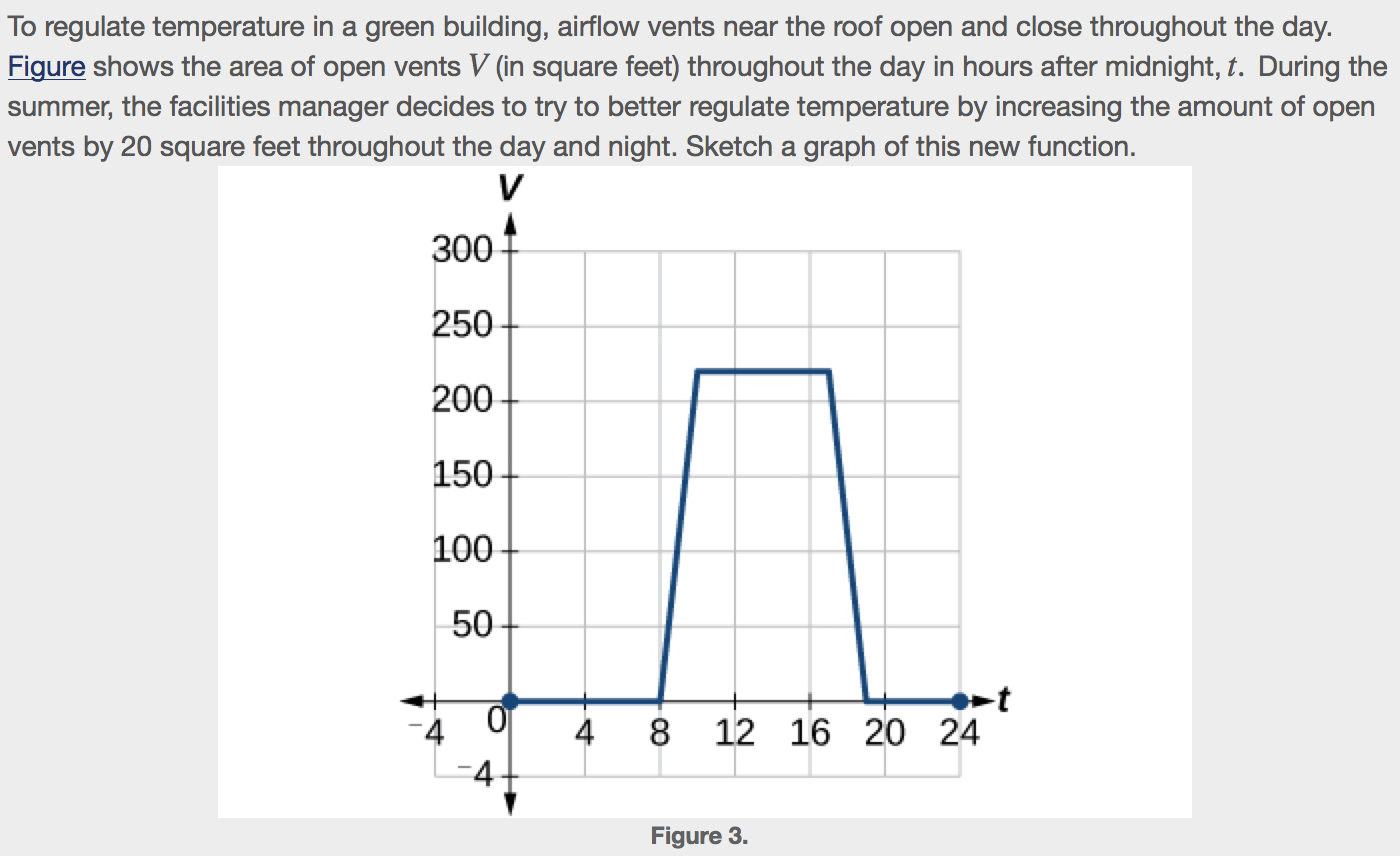
**Identifying Vertical Shifts**

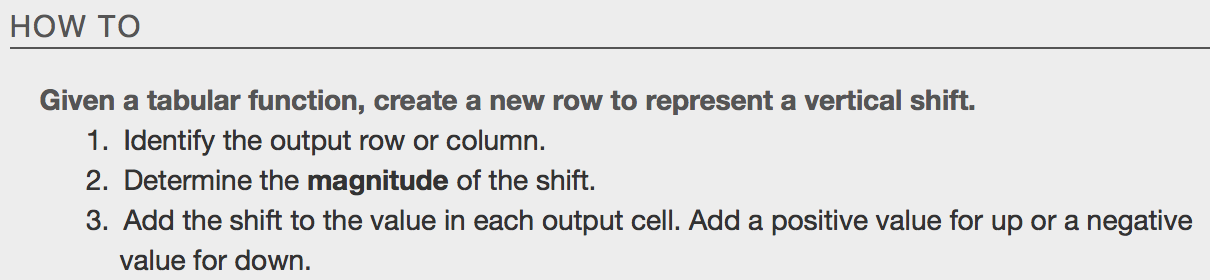
One simple kind of transformation involves shifting the entire graph of a function up, down, right, or left. The simplest shift is a **vertical shift**, moving the graph up or down, because this transformation involves adding a positive or negative constant to the function. In other words, we add the same constant to the output value of the function regardless of the input. For a function *g*(*x*)=*f*(*x*)+*k*,the function *f*(*x*)is shifted vertically *k* units. See [Figure](http://cnx.org/contents/E6wQevFf@5.241:X2_wKhAA@7/Transformation-of-Functions#Figure_01_05_002) for an example.





Example

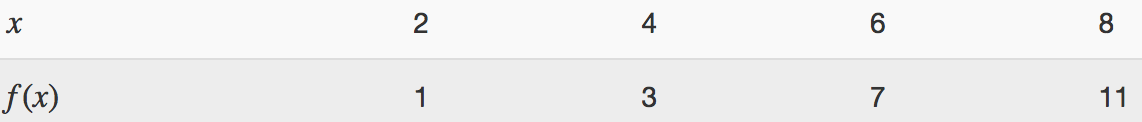




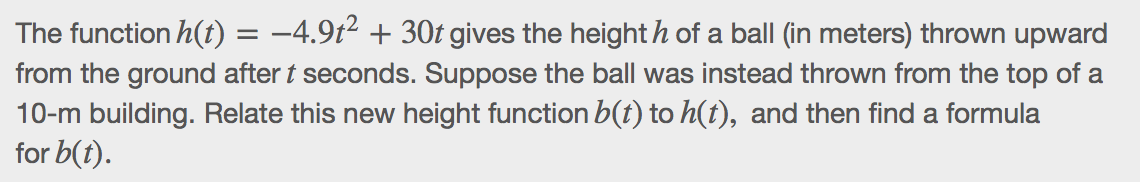
**Example**

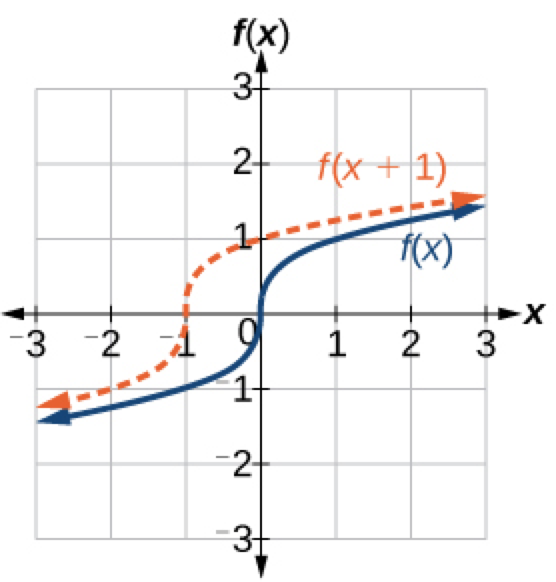
**a.**

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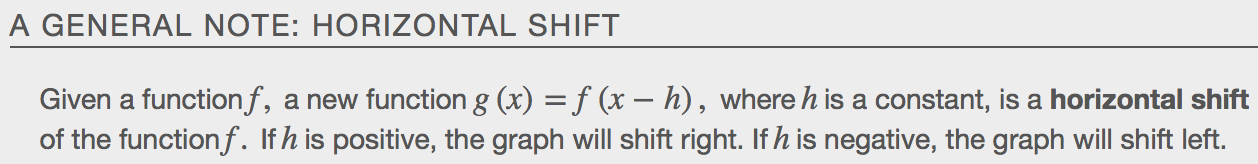
**b.**

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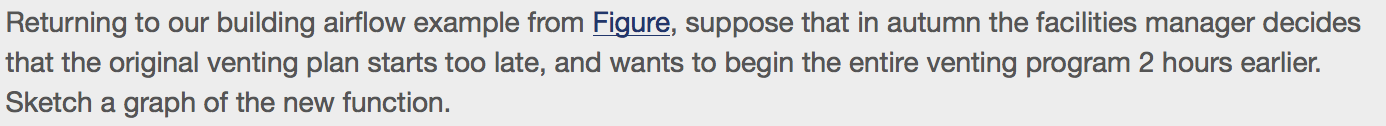
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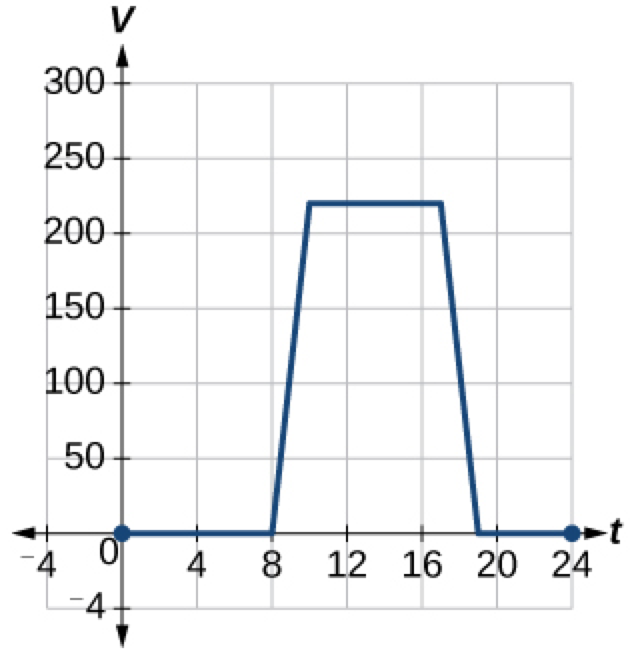
**Identifying Horizontal Shifts**

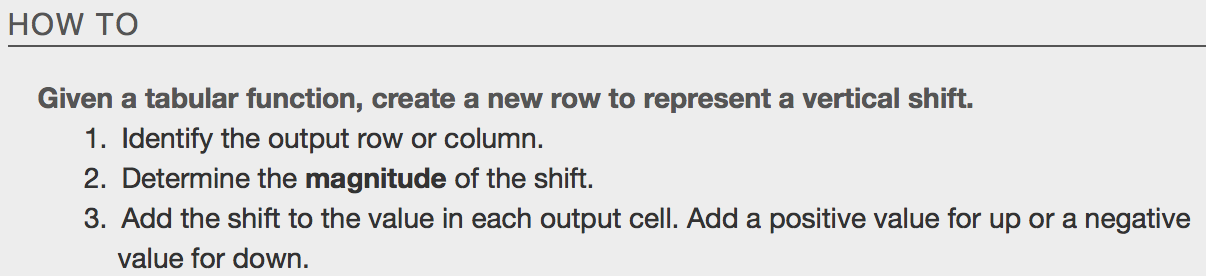
We just saw that the vertical shift is a change to the output, or outside, of the function. We will now look at how changes to input, on the inside of the function, change its graph and meaning. A shift to the input results in a movement of the graph of the function left or right in what is known as a **horizontal shift**, shown in [Figure](http://cnx.org/contents/E6wQevFf@5.241:X2_wKhAA@7/Transformation-of-Functions#Figure_01_05_005).

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**Example**

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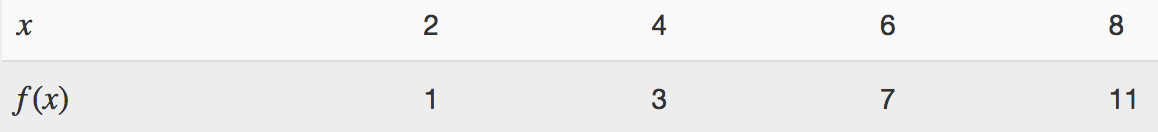
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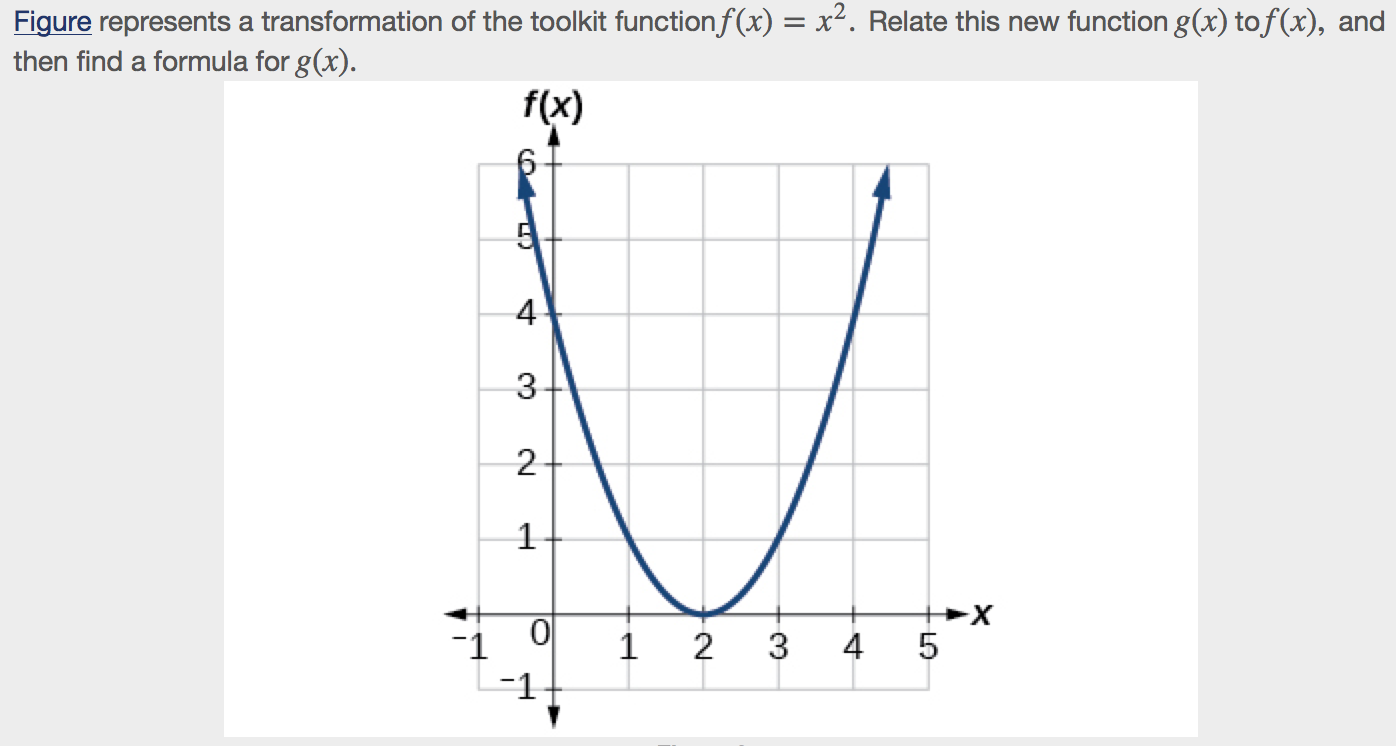
**Example**

**a.**

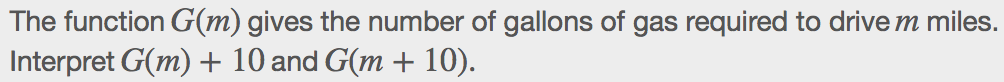
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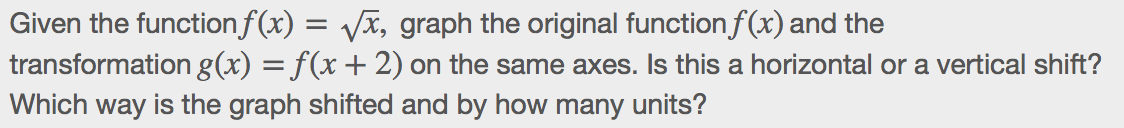
**b.**

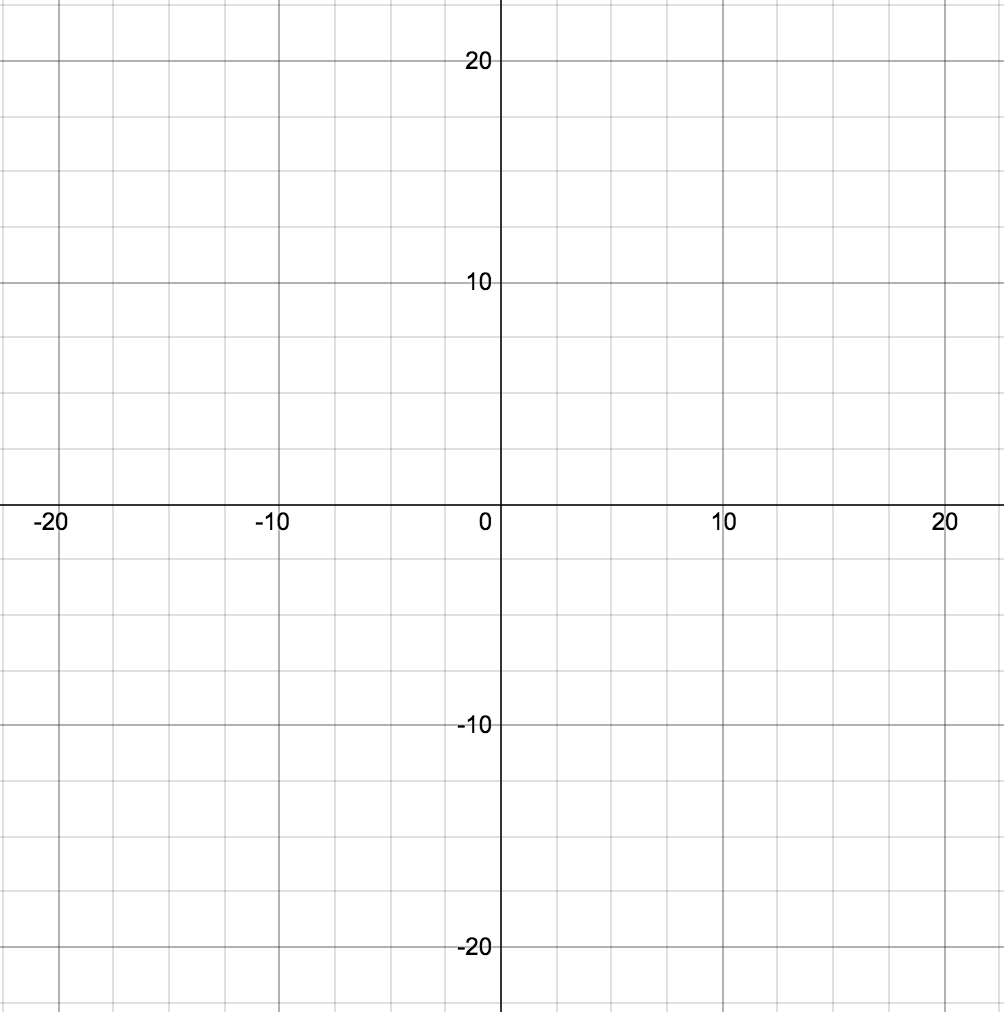
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**c.**

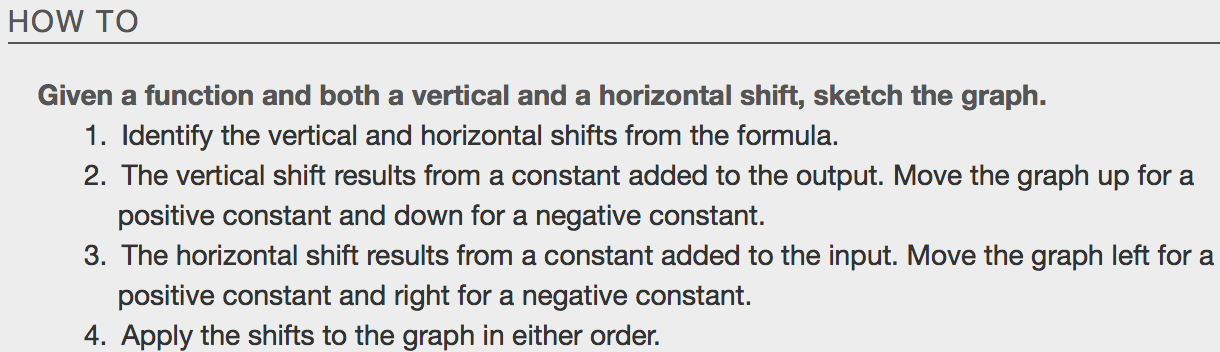
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**d.**

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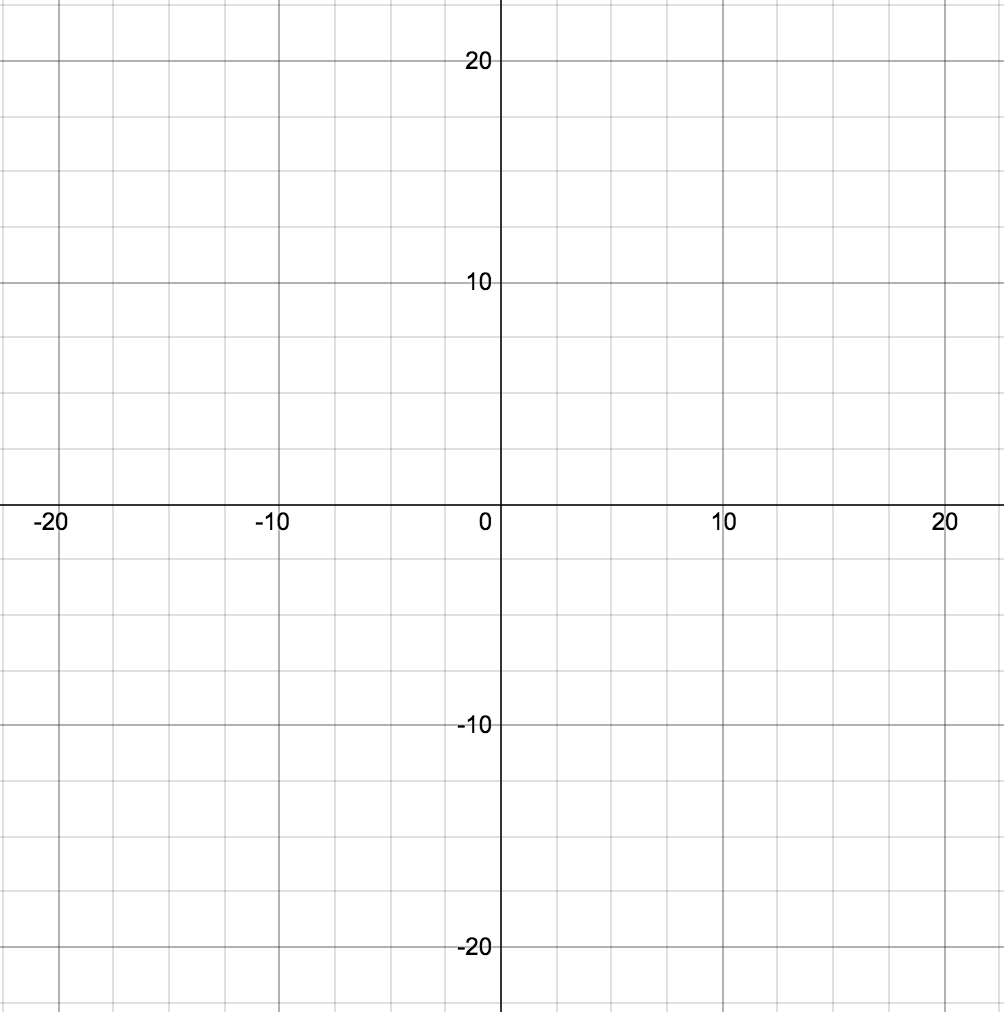
**Combining Vertical and Horizontal Shifts**

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**Example**

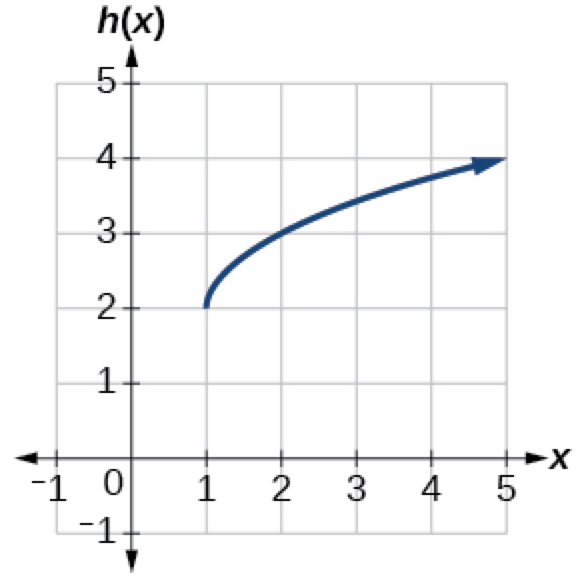
**a.**

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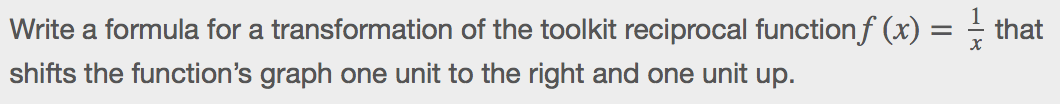
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**b.**

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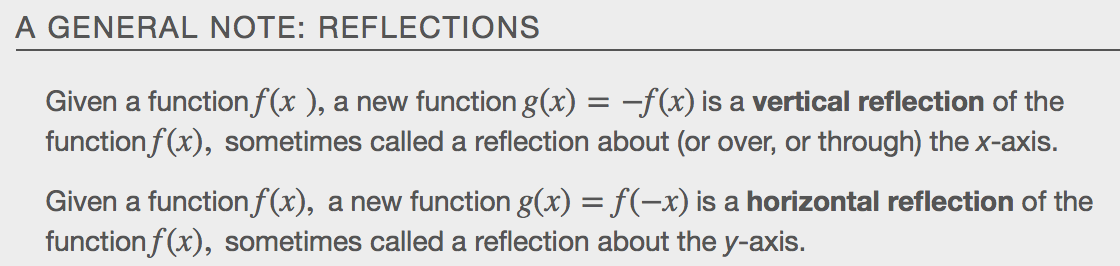
**c.**

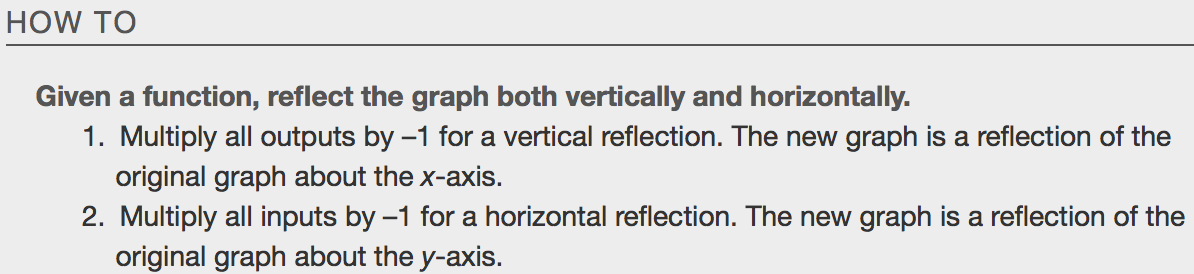
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**Graphing Functions Using Reflections About Axes**

Another transformation that can be applied to a function is a reflection over the *x*- or *y*-axis. A **vertical reflection** reflects a graph vertically across the *\_\_\_\_*-axis, while a **horizontal reflection** reflects a graph horizontally across the *\_\_\_\_*-axis. The reflections are shown in [Figure](http://cnx.org/contents/E6wQevFf@5.241:X2_wKhAA@7/Transformation-of-Functions#Figure_01_05_013).



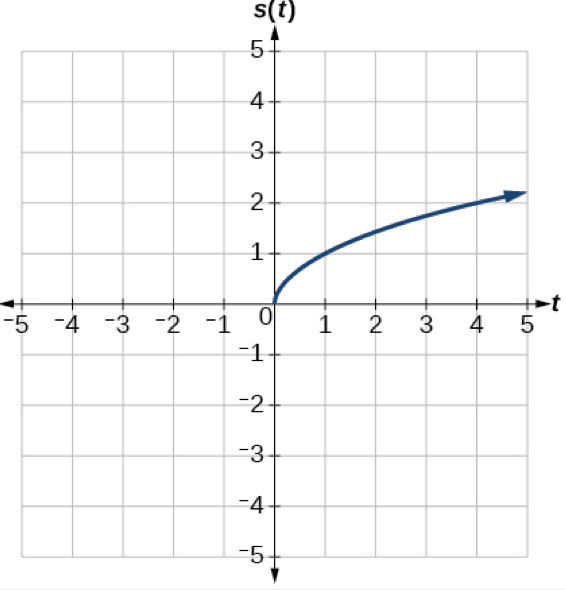
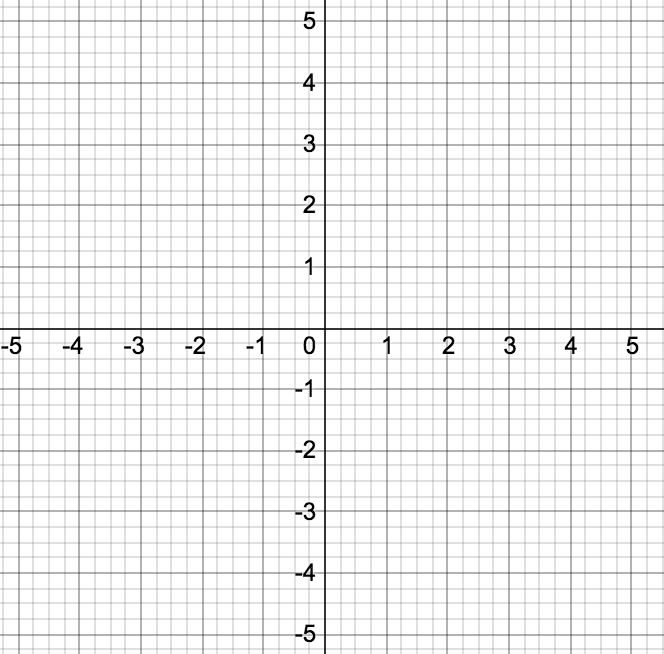
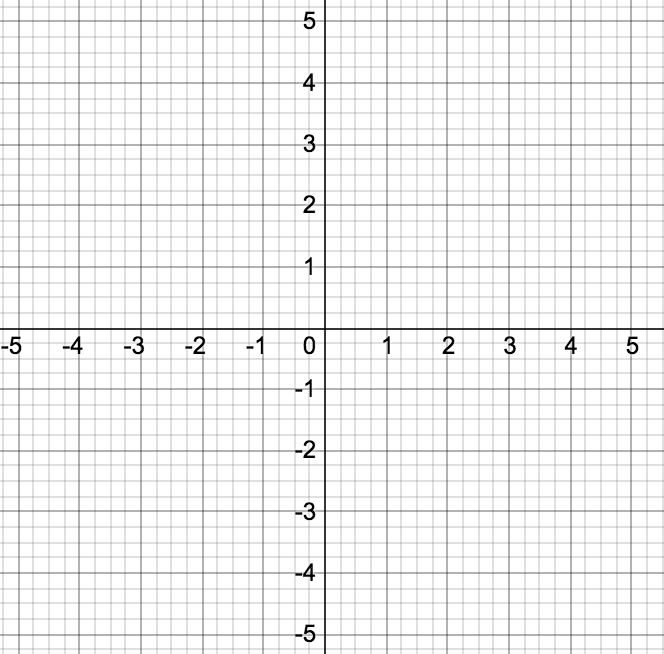




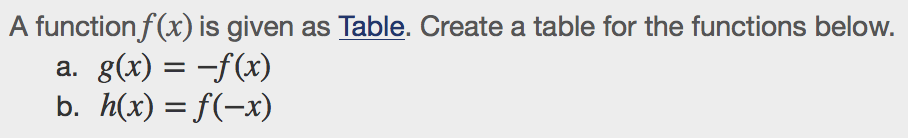
**Example**

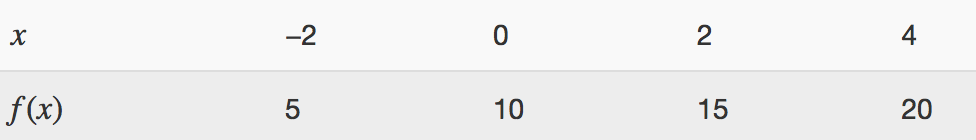
**a.**

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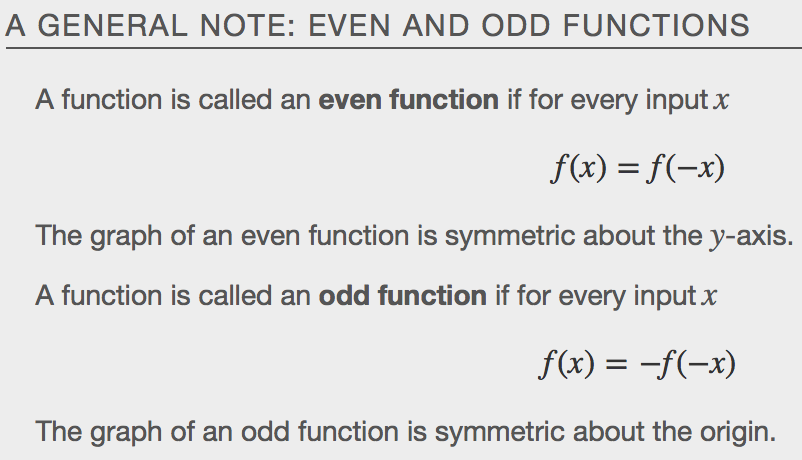
**  **

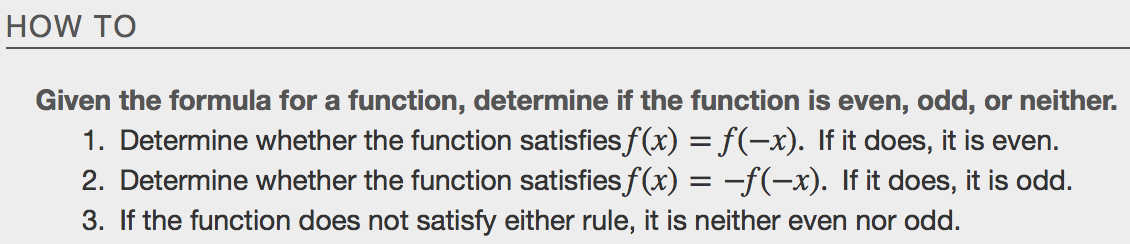
**b.**

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**Even and Odd Functions**

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**Example**

**a. **

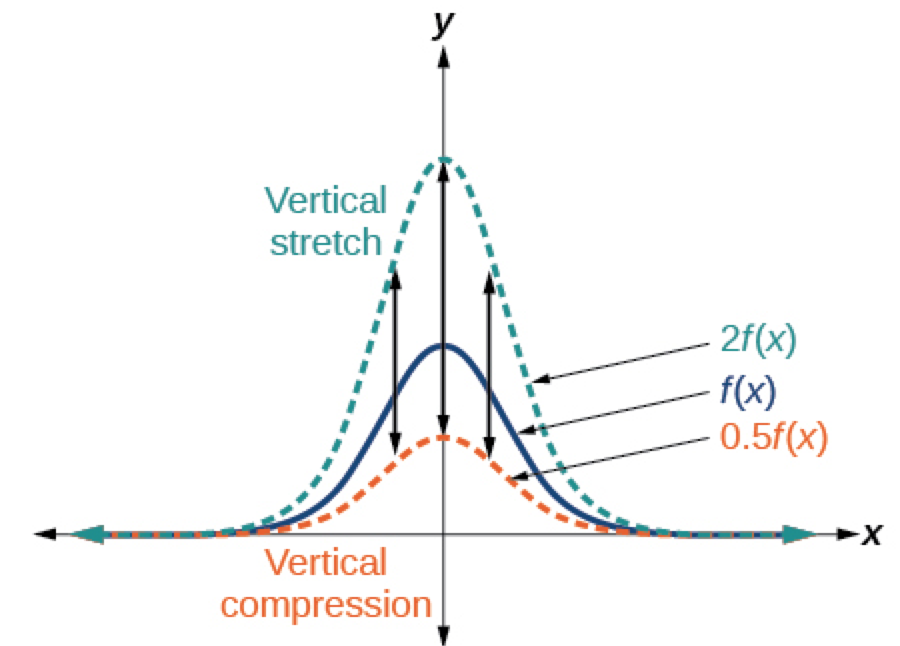
**b. **

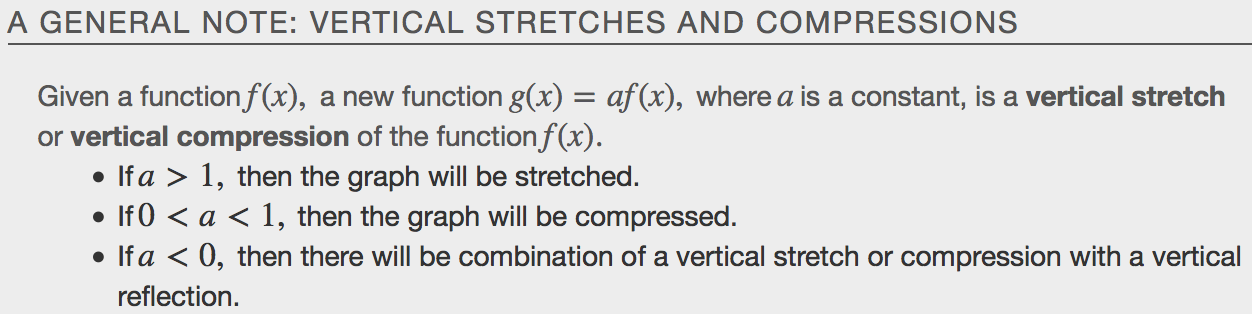
**Graphing Functions Using Stretches and Compressions**

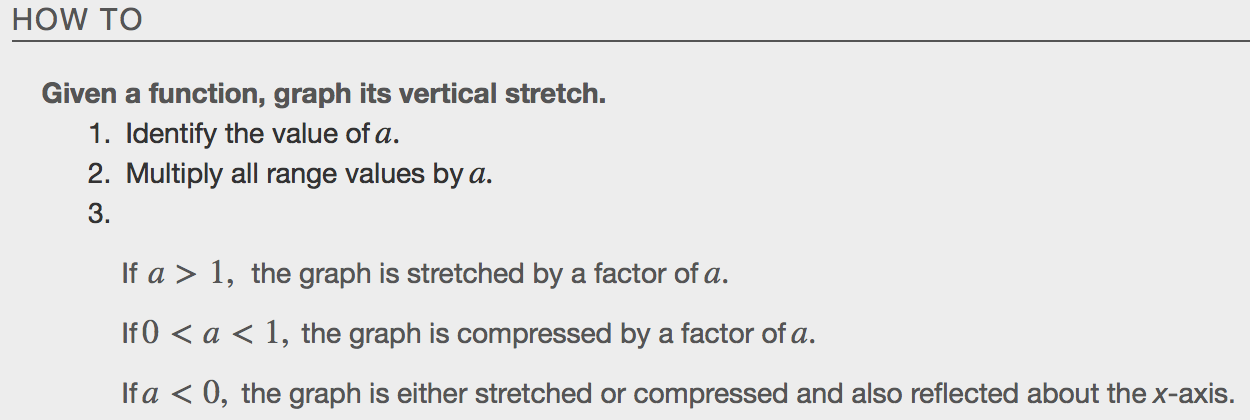
Adding a constant to the inputs or outputs of a function changed the position of a graph with respect to the axes, but it did not affect the shape of a graph. We now explore the effects of multiplying the inputs or outputs by some quantity. We can transform the inside (input values) of a function or we can transform the outside (output values) of a function. Each change has a specific effect that can be seen graphically.

**Vertical Stretches and Compressoins**

When we multiply a function by a positive constant, we get a function whose graph is stretched or compressed vertically in relation to the graph of the original function. If the constant is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ than 1, we get a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**; if the constant is between \_\_\_ and \_\_\_ , we get a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**. [Figure](http://cnx.org/contents/E6wQevFf@5.241:X2_wKhAA@7/Transformation-of-Functions#Figure_01_05_025) shows a function multiplied by constant factors 2 and 0.5 and the resulting vertical stretch and compression.



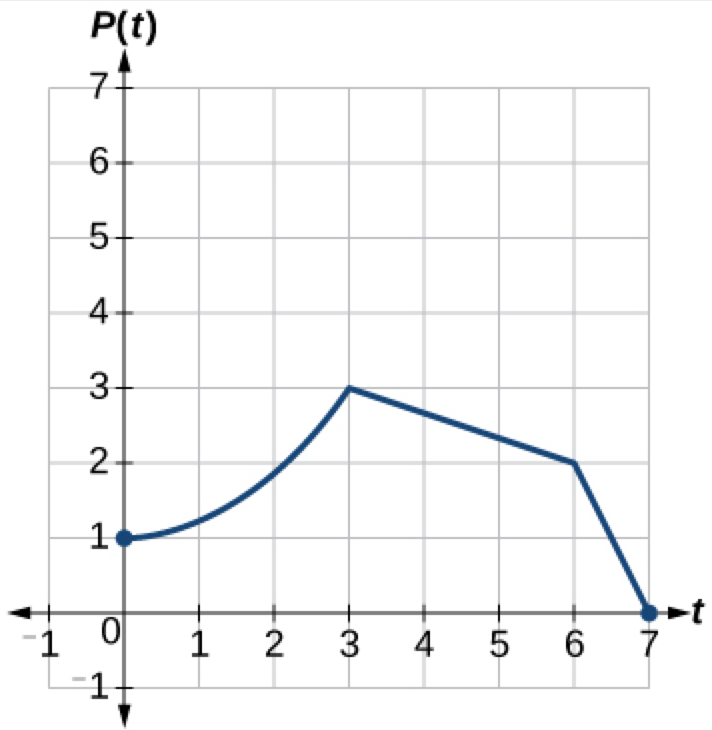


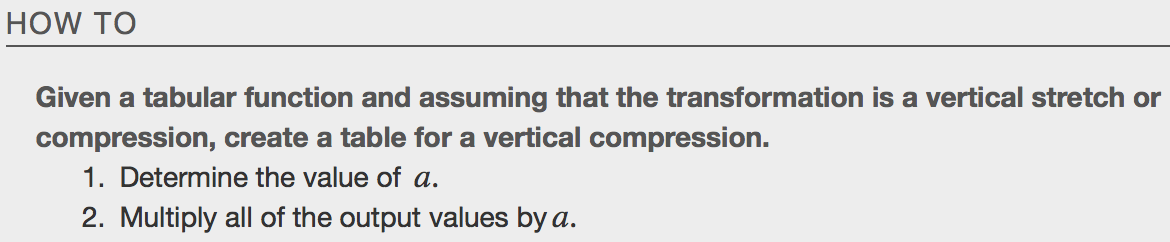


**Example**

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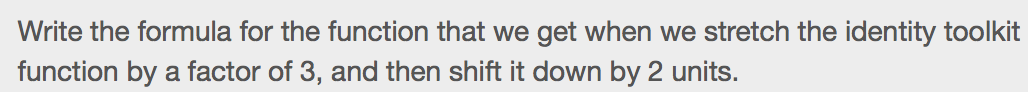
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**Example**

**a.**

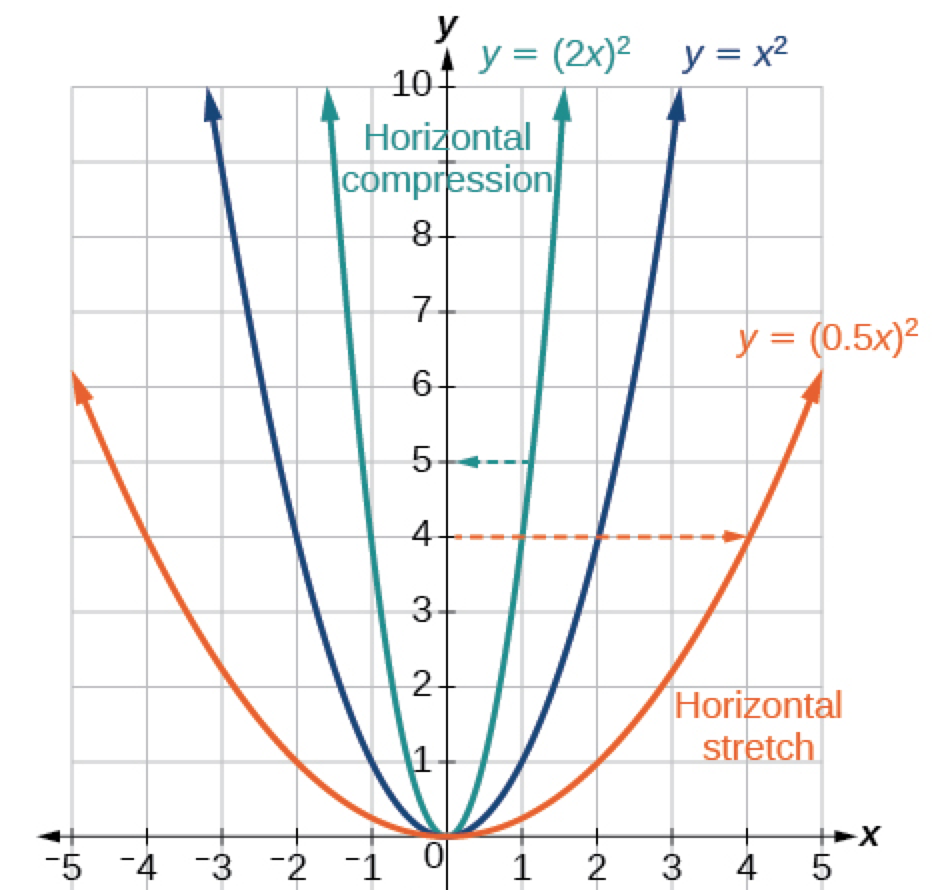
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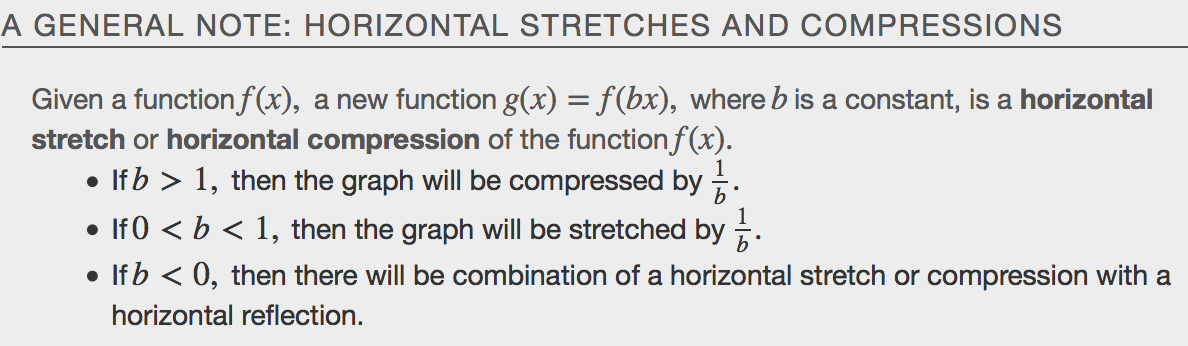
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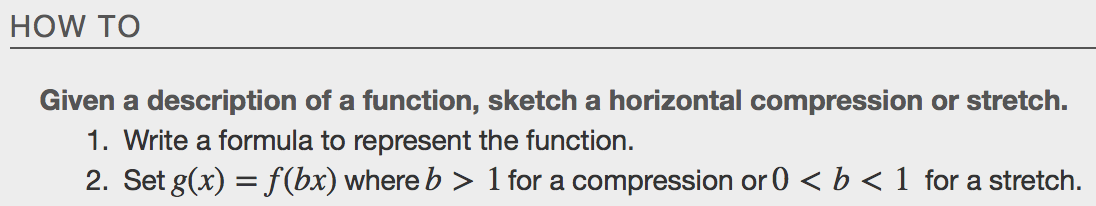
**b. **

**Horizontal Stretches and Compressions**

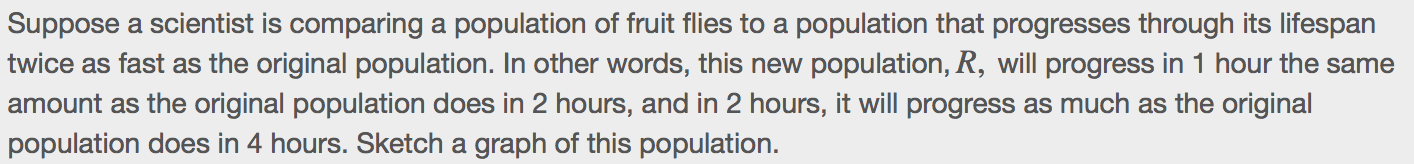
Now we consider changes to the inside of a function. When we multiply a function’s input by a positive constant, we get a function whose graph is stretched or compressed horizontally in relation to the graph of the original function. If the constant is between \_\_\_\_\_ and \_\_\_\_\_, we get a **horizontal \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**; if the constant is greater than \_\_\_\_\_, we get a **horizontal \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** of the function.

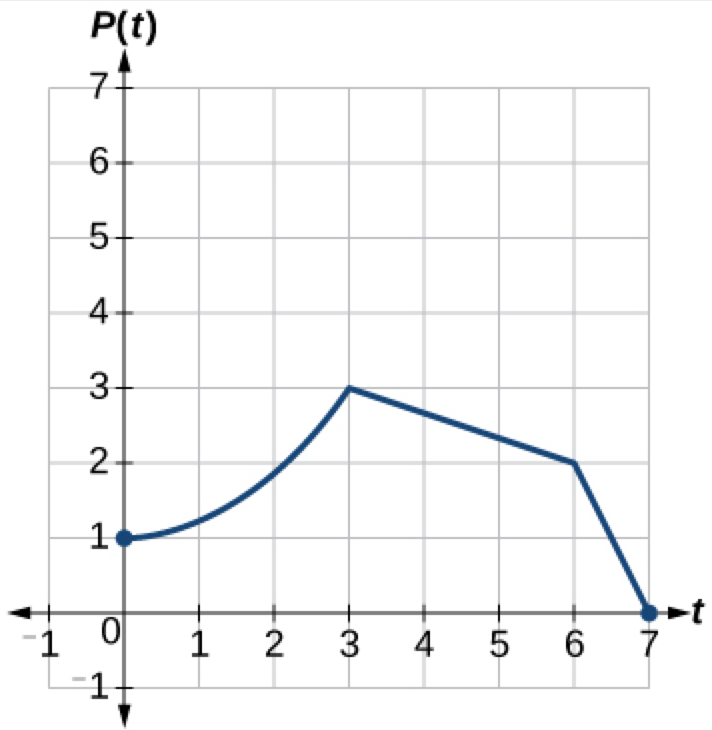


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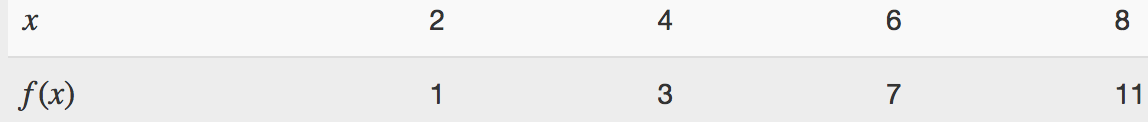
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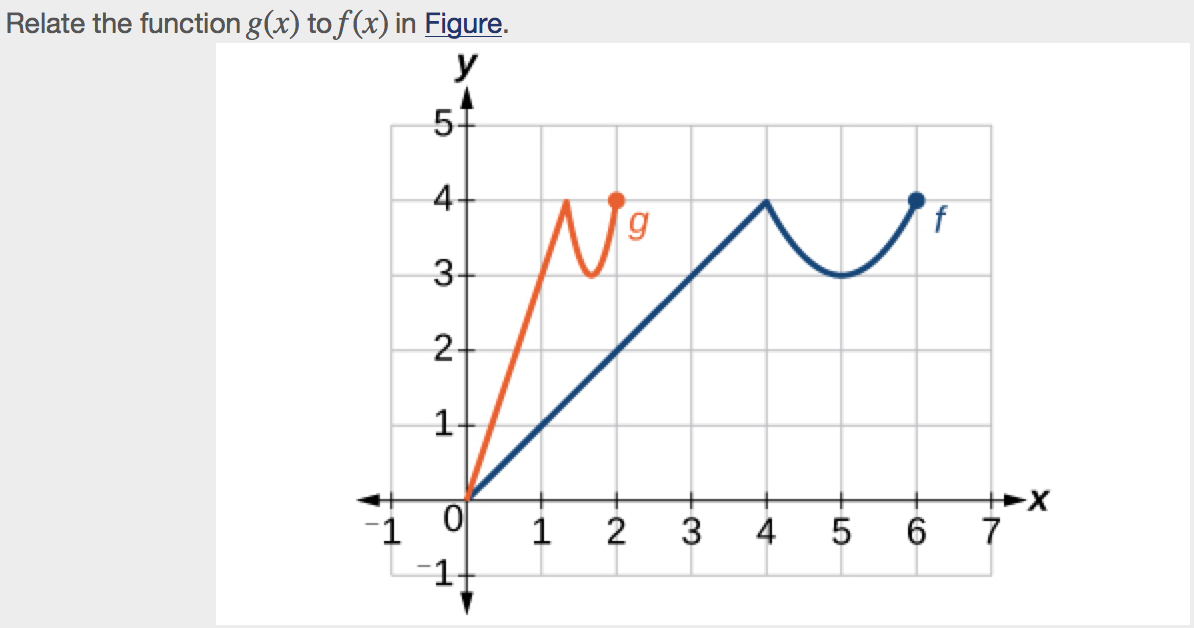
**Example**

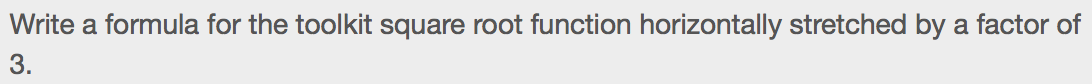
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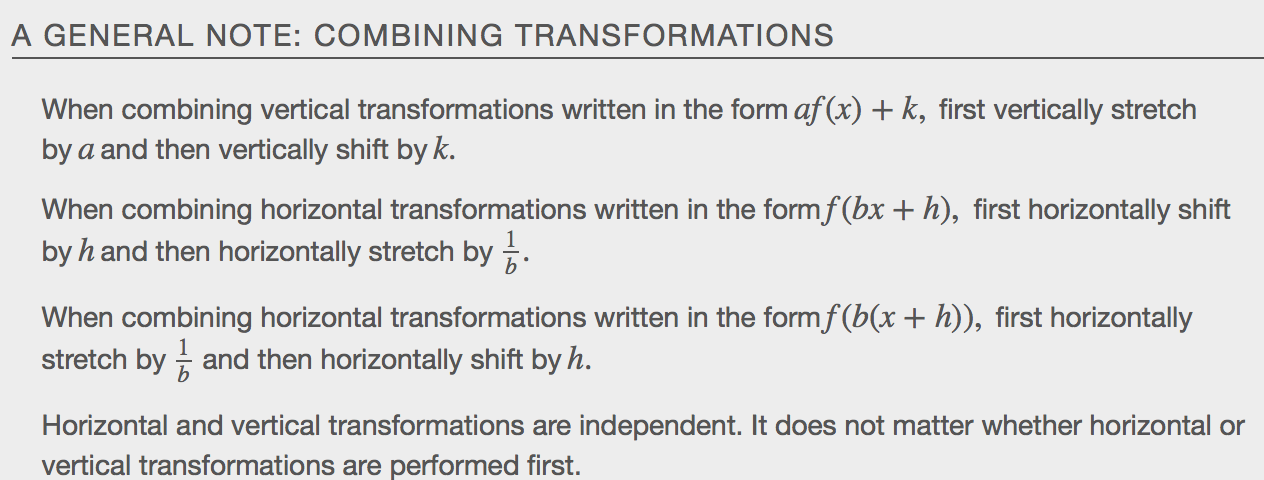
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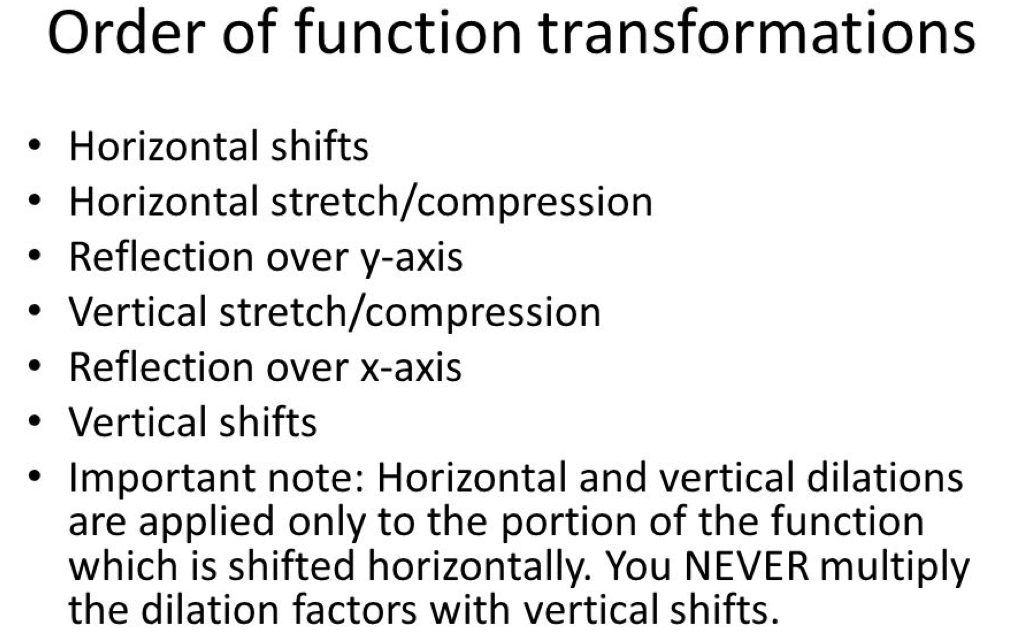
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**Performing a Series of Transformations**

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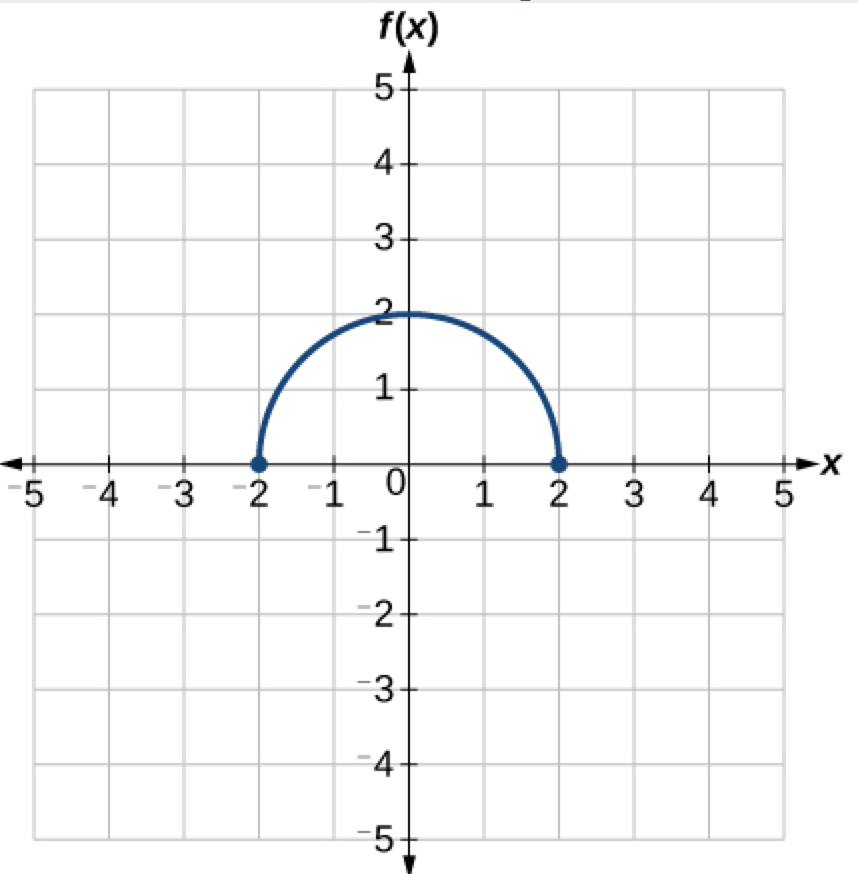


**Example**

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