3.4 – Composition of Functions

Function ______________________________________ is only one way to combine existing functions. Another way is to carry out the usual algebraic operations on functions, such as addition, subtraction, multiplication and division. We do this by performing the operations with the function outputs, defining the result as the output of our new function.

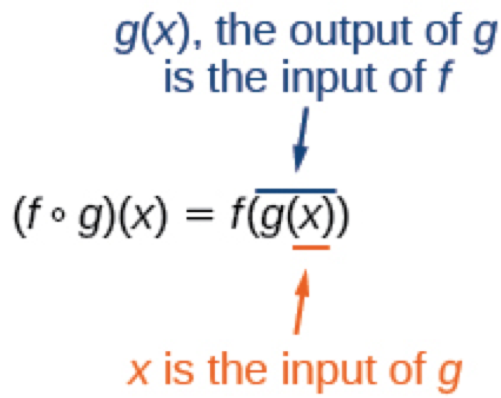
A GENERAL NOTE: COMPOSITION OF FUNCTIONS

When the output of one function is used as the input of another, we call the entire operation a composition of functions. For any input x and functions f and g, this action defines a **composite function**, which we write as $f \circ g$ such that

$$(f \circ g)(x) = f(g(x))$$

The domain of the composite function $f \circ g$ is all x such that x is in the domain of g and $g(x)$ is in the domain of f.

It is important to realize that the product of functions fg is not the same as the function composition $f(g(x))$, because, in general, $f(x)g(x) \neq f(g(x))$.



Example

a.

Using the functions provided, find $f(g(x))$ and $g(f(x))$. Determine whether the composition of the functions is **commutative**.

$$f(x) = 2x + 1 \qquad g(x) = 3 - x$$

b.

The function $c(s)$ gives the number of calories burned completing s sit-ups, and $s(t)$ gives the number of sit-ups a person can complete in t minutes. Interpret $c(s(3))$.

c.

Suppose $f(x)$ gives miles that can be driven in x hours and $g(y)$ gives the gallons of gas used in driving y miles. Which of these expressions is meaningful: $f(g(y))$ or $g(f(x))$?

Q&A: ARE THERE ANY SITUATIONS WHERE $f(g(y))$ AND $g(f(x))$ WOULD BOTH BE MEANINGFUL OR USEFUL EXPRESSIONS?

Yes. For many pure mathematical functions, both compositions make sense, even though they usually produce different new functions. In real-world problems, functions whose inputs and outputs have the same units also may give compositions that are meaningful in either order.

Evaluating Composite Functions

Once we compose a new function from two existing functions, we need to be able to evaluate it for any input in its ___________________________. We will do this with specific numerical inputs for functions expressed as tables, graphs, and formulas and with variables as inputs to functions expressed as formulas. In each case, we evaluate the ________________________ function using the starting input and then use the inner function's __________________ as the __________________ for the outer function.

Evaluating Composite Functions Using Tables

Once we compose a new function from two existing functions, we need to be able to evaluate it for any input in its domain. We will do this with specific numerical inputs for functions expressed as tables, graphs, and formulas and with variables as inputs to functions expressed as formulas. In each case, we evaluate the inner function using the starting _________________and then use the inner function's _______________ as the ____________________for the outer function.

Example

Using Table, evaluate $f(g(1))$ and $g(f(4))$.

x	$f(x)$	$g(x)$
1	6	3
2	8	5
3	3	2
4	1	7

Evaluating Composite Functions Using Graphs

When we are given individual functions as graphs, the procedure for evaluating composite functions is similar to the process we use for evaluating tables. We read the input and output values, but this time, from the x- and y-axes of the graphs.

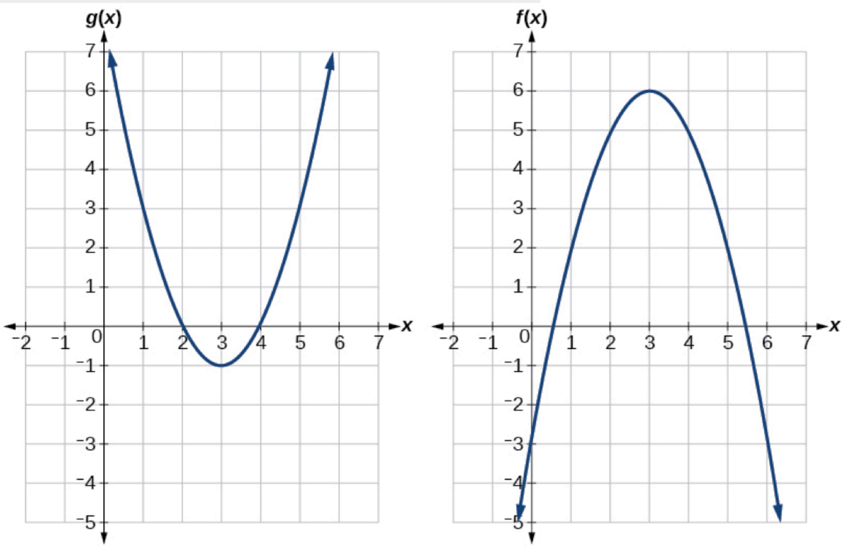
HOW TO

Given a composite function and graphs of its individual functions, evaluate it using the information provided by the graphs.

1. Locate the given input to the inner function on the x-axis of its graph.
2. Read off the output of the inner function from the y-axis of its graph.
3. Locate the inner function output on the x-axis of the graph of the outer function.
4. Read the output of the outer function from the y-axis of its graph. This is the output of the composite function.

Example

Using Figure, evaluate $g(f(2))$.



(a) (b)

Evaluating Composite Functions Using Formulas

When evaluating a composite function where we have either created or been given formulas, the rule of working from the inside out remains the same. The input value to the outer function will be the output of the inner function, which may be a numerical value, a variable name, or a more complicated expression

HOW TO

Given a formula for a composite function, evaluate the function.

1. Evaluate the inside function using the input value or variable provided.
2. Use the resulting output as the input to the outside function.

Example

Given $f(t) = t^2 - t$ and $h(x) = 3x + 2$, evaluate

a. $h(f(2))$

b. $h(f(-2))$

Finding the Domain of a Composite Function

A GENERAL NOTE: DOMAIN OF A COMPOSITE FUNCTION

The domain of a composite function $f(g(x))$ is the set of those inputs x in the domain of g for which $g(x)$ is in the domain of f.

HOW TO

Given a function composition $f(g(x))$, determine its domain.

1. Find the domain of g.
2. Find the domain of f.
3. Find those inputs x in the domain of g for which $g(x)$ is in the domain of f. That is, exclude those inputs x from the domain of g for which $g(x)$ is not in the domain of f. The resulting set is the domain of $f \circ g$.

Examples

a.

Find the domain of

$$(f \circ g)(x) \quad \text{where} \quad f(x) = \frac{5}{x-1} \quad \text{and} \quad g(x) = \frac{4}{3x-2}$$

b.

Find the domain of

$$(f \circ g)(x) \text{ where } f(x) = \sqrt{x+2} \text{ and } g(x) = \sqrt{3-x}$$

c.

Find the domain of

$$(f \circ g)(x) \text{ where } f(x) = \frac{1}{x-2} \text{ and } g(x) = \sqrt{x+4}$$

Decomposing a Composite Function

In some cases, it is necessary to ________________________________ a complicated function. In other words, we can write it as a composition of __________ simpler functions. There may be ____________ than __________ way to decompose a composite function, so we may choose the decomposition that appears to be most expedient.

Example

a. Write $f(x) = \sqrt{5-x^2}$ as the composition of two functions.

b. Write $f(x) = \frac{4}{3-\sqrt{4+x^2}}$ as the composition of two functions.