A GENERAL NOTE: RATE OF CHANGE

A rate of change describes how an output quantity changes relative to the change in the input quantity. The units on a rate of change are "output units per input units."

The average rate of change between two input values is the total change of the function values (output values) divided by the change in the input values.

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Some examples of rates of change include:

- A population of rats increasing by 40 rats per week
- A car traveling 68 miles per hour (distance traveled changes by 68 miles each hour as time passes)
- A car driving 27 miles per gallon (distance traveled changes by 27 miles for each gallon)
- The current through an electrical circuit increasing by 0.125 amperes for every volt of increased voltage
- The amount of money in a college account decreasing by $4,000 per quarter

HOW TO

Given the value of a function at different points, calculate the average rate of change of a function for the interval between two values x_1 and x_2.

1. Calculate the difference $y_2 - y_1 = \Delta y$.
2. Calculate the difference $x_2 - x_1 = \Delta x$.
3. Find the ratio $\frac{\Delta y}{\Delta x}$.

Example:

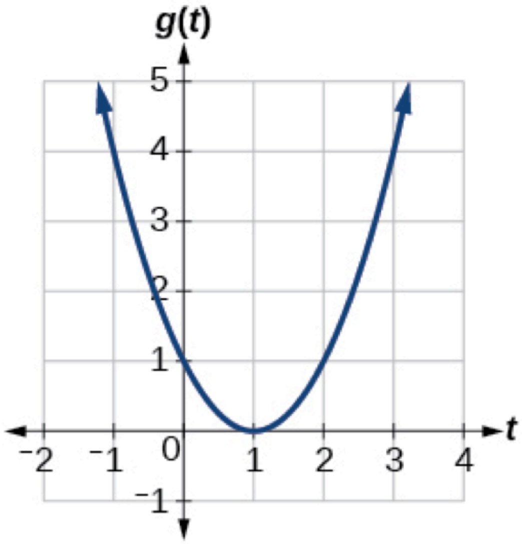
a. Gasoline costs have experienced some wild fluctuations over the last several decades. Table[1] lists the average cost, in dollars, of a gallon of gasoline for the years 2005–2012. The cost of gasoline can be considered as a function of year.

y	2005	2006	2007	2008	2009	2010	2011	2012
$C(y)$	2.31	2.62	2.84	3.30	2.41	2.84	3.58	3.68

Using the data in Table, find the average rate of change between 2005 and 2010.

b.

Given the function $g(t)$ shown in Figure, find the average rate of change on the interval $[-1, 2$



c.

After picking up a friend who lives 10 miles away and leaving on a trip, Anna records her distance from home over time. values are shown in Table. Find her average speed over the first 6 hours.

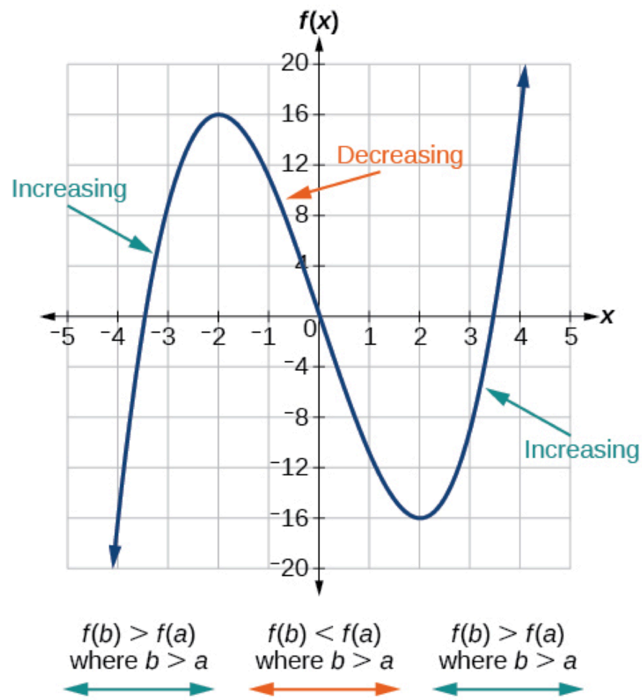
***t* (hours)**	0	1	2	3	4	5	6	7
***D*(*t*) (miles)**	10	55	90	153	214	240	292	3(

d.

Find the average rate of change of $f(x) = x - 2\sqrt{x}$ on the interval $[1, 9]$.

Using a Graph to Determine Where a Function is Increasing, Decreasing, or Constant

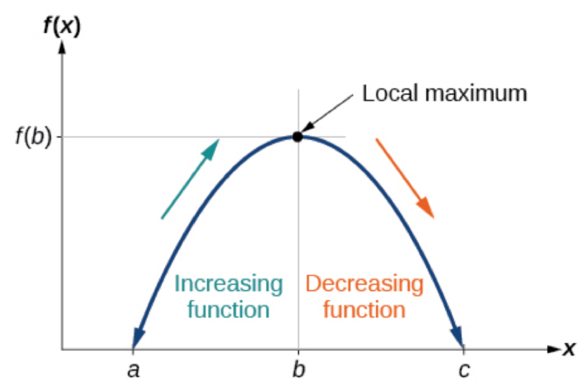
As part of exploring how functions change, we can identify intervals over which the function is changing in specific ways. We say that a function is ____________________________ on an interval if the function values increase as the input values increase within that interval. Similarly, a function is _________________________ on an interval if the function values decrease as the input values increase over that interval. The average rate of change of an increasing function is positive, and the average rate of change of a decreasing function is negative.



Local maxima:

Local minima:

Local Extrema:



A GENERAL NOTE: LOCAL MINIMA AND LOCAL MAXIMA

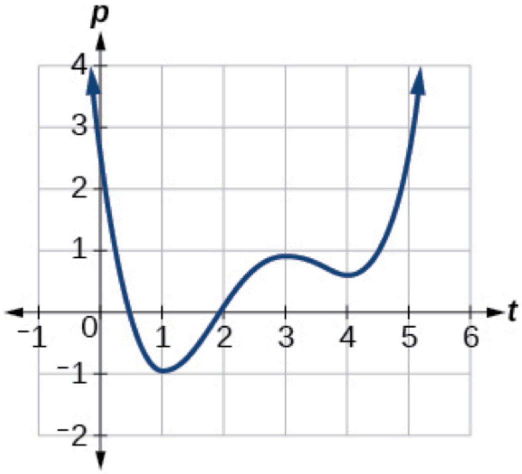
A function f is an **increasing function** on an open interval if $f(b) > f(a)$ for any two input values a and b in the given interval where $b > a$.

A function f is a **decreasing function** on an open interval if $f(b) < f(a)$ for any two input values a and b in the given interval where $b > a$.

A function f has a local maximum at $x = b$ if there exists an interval (a, c) with $a < b < c$ such that, for any x in the interval (a, c), $f(x) \leq f(b)$. Likewise, f has a local minimum at $x = b$ if there exists an interval (a, c) with $a < b < c$ such that, for any x in the interval (a, c), $f(x) \geq f(b)$.

Examples

a. Given the function $p(t)$ in Figure, identify the intervals on which the function appears to be increasing and decreasing.



b. Graph the function $f(x) = x^3 - 6x^2 - 15x + 20$ to estimate the local extrema of the function. Use these to determine the intervals on which the function is increasing and decreasing.

→ TI-83 or TI-84 calculator or www.desmos.com

A GENERAL NOTE: ABSOLUTE MAXIMA AND MINIMA

The **absolute maximum** of f at $x = c$ is $f(c)$ where $f(c) \geq f(x)$ for all x in the domain of f.

The **absolute minimum** of f at $x = d$ is $f(d)$ where $f(d) \leq f(x)$ for all x in the domain of f.

c. Identify the following for the graph given.

a. Average rate of change from x=1 to x=4

b. Increasing intervals

c. Decreasing Intervals

d. All local extrema

e. Absolute maximum and absolute minimum (if possible)