

3.2 – Domain and Range

In creating various functions using the data, we can identify different independent and dependent variables, and we can analyze the data and the functions to determine the domain and range. In this section, we will investigate methods for determining the domain and range of functions such as these.

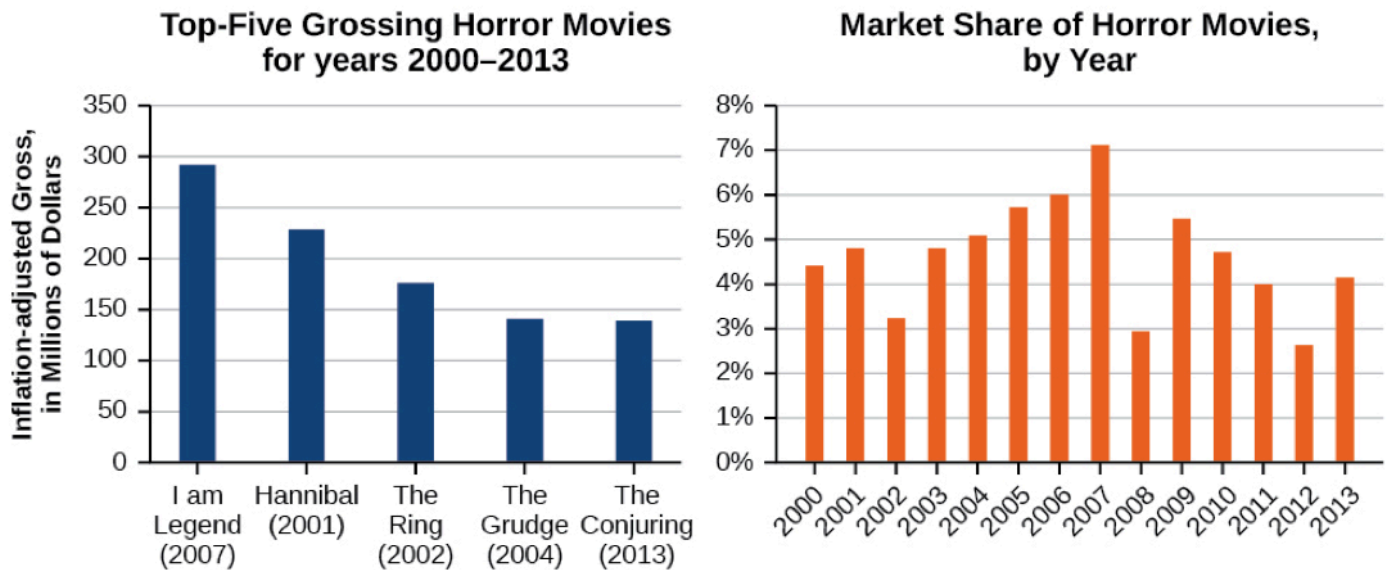
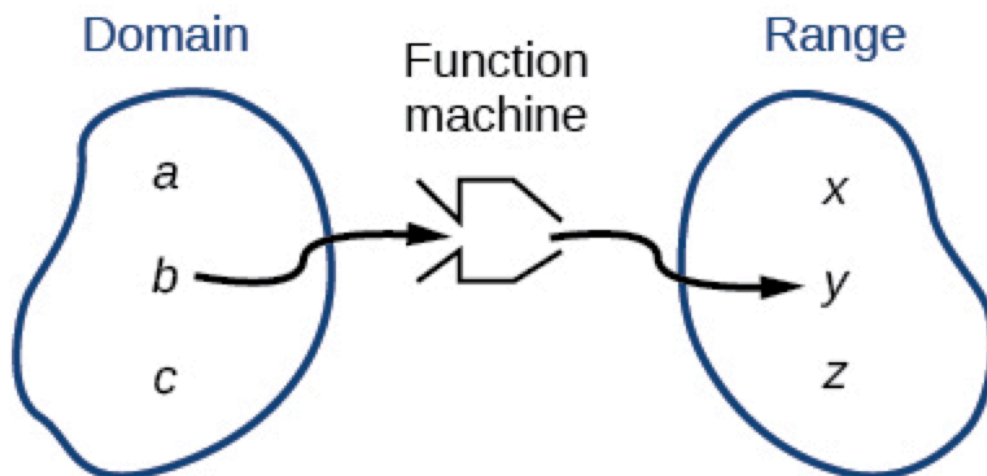


Figure 1. Based on data compiled by www.the-numbers.com.¹

What do we need to consider when identifying domain and range?

- _____ roots of _____ numbers
- Division by _____
- Combination of both.

We can visualize the domain as a “holding area” that contains “raw materials” for a “function machine” and the range as another “holding area” for the machine’s products. See [Figure](#).



****We write domain and range as what IS possible, excluding values that are not. It is written in _____ notation. ***

Convention of Interval Notation

- The smallest number from the interval is written _____.
- The largest number in the interval is written second, following a _____.
- Parentheses, (or), are used to signify that an endpoint value is _____ included, called _____.
- Brackets, [or], are used to indicate that an endpoint value _____ included, called _____.
- Infinity _____ and negative infinity _____ are NEVER included. (Always use _____)









Inequality	Interval Notation	Graph on Number Line	Description
$x > a$	(a, ∞)		x is greater than a
$x < a$	$(-\infty, a)$		x is less than a
$x \geq a$	$[a, \infty)$		x is greater than or equal to a
$x \leq a$	$(-\infty, a]$		x is less than or equal to a
$a < x < b$	(a, b)		x is strictly between a and b
$a \leq x < b$	$[a, b)$		x is between a and b , to include a
$a < x \leq b$	$(a, b]$		x is between a and b , to include b
$a \leq x \leq b$	$[a, b]$		x is between a and b , to include a and b

Figure 3.

Example:

Find the domain of the function:

$$\{(-5, 4), (0, 0), (5, -4), (10, -8), (15, -12)\}$$

HOW TO

Given a function written in equation form, find the domain.

1. Identify the input values.
2. Identify any restrictions on the input and exclude those values from the domain.
3. Write the domain in interval form, if possible.

Examples:

Finding the Domain of a Function

a. Find the domain of the function $f(x) = x^2 - 1$.

Find the domain of the function: $f(x) = 5 - x + x^3$.

b.

HOW TO

Given a function written in an equation form that includes a fraction, find the domain.

1. Identify the input values.
2. Identify any restrictions on the input. If there is a denominator in the function's formula, set the denominator equal to zero and solve for x . If the function's formula contains an even root, set the radicand greater than or equal to 0, and then solve.
3. Write the domain in interval form, making sure to exclude any restricted values from the domain.

Example

Find the domain of the function: $f(x) = \frac{1+4x}{2x-1}$.

HOW TO

Given a function written in equation form including an even root, find the domain.

1. Identify the input values.
2. Since there is an even root, exclude any real numbers that result in a negative number in the radicand. Set the radicand greater than or equal to zero and solve for x .
3. The solution(s) are the domain of the function. If possible, write the answer in interval form.

Example

Find the domain of the function $f(x) = \sqrt{5 + 2x}$.

Other Notations for Domain and Range






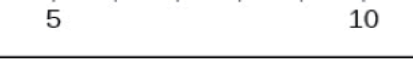
	Inequality Notation	Set-builder Notation	Interval Notation
	$5 < h \leq 10$	$\{h \mid 5 < h \leq 10\}$	$(5, 10]$
	$5 \leq h < 10$	$\{h \mid 5 \leq h < 10\}$	$[5, 10)$
	$5 < h < 10$	$\{h \mid 5 < h < 10\}$	$(5, 10)$
	$h < 10$	$\{h \mid h < 10\}$	$(-\infty, 10)$
	$h \geq 10$	$\{h \mid h \geq 10\}$	$[10, \infty)$
	All real numbers	\mathbb{R}	$(-\infty, \infty)$

Figure 4.

Example

Given [Figure](#), specify the graphed set in

- words
- set-builder notation
- interval notation



Figure 6.

Identifying Domain and Range from a Graph

Another way to identify the domain and range of functions is by using graphs. Because the domain refers to the set of possible input values, the domain of a graph consists of all the input values shown on the x -axis. The range is the set of possible output values, which are shown on the y -axis. Keep in mind that if the graph continues beyond the portion of the graph we can see, the domain and range may be greater than the visible values. See [Figure](#).

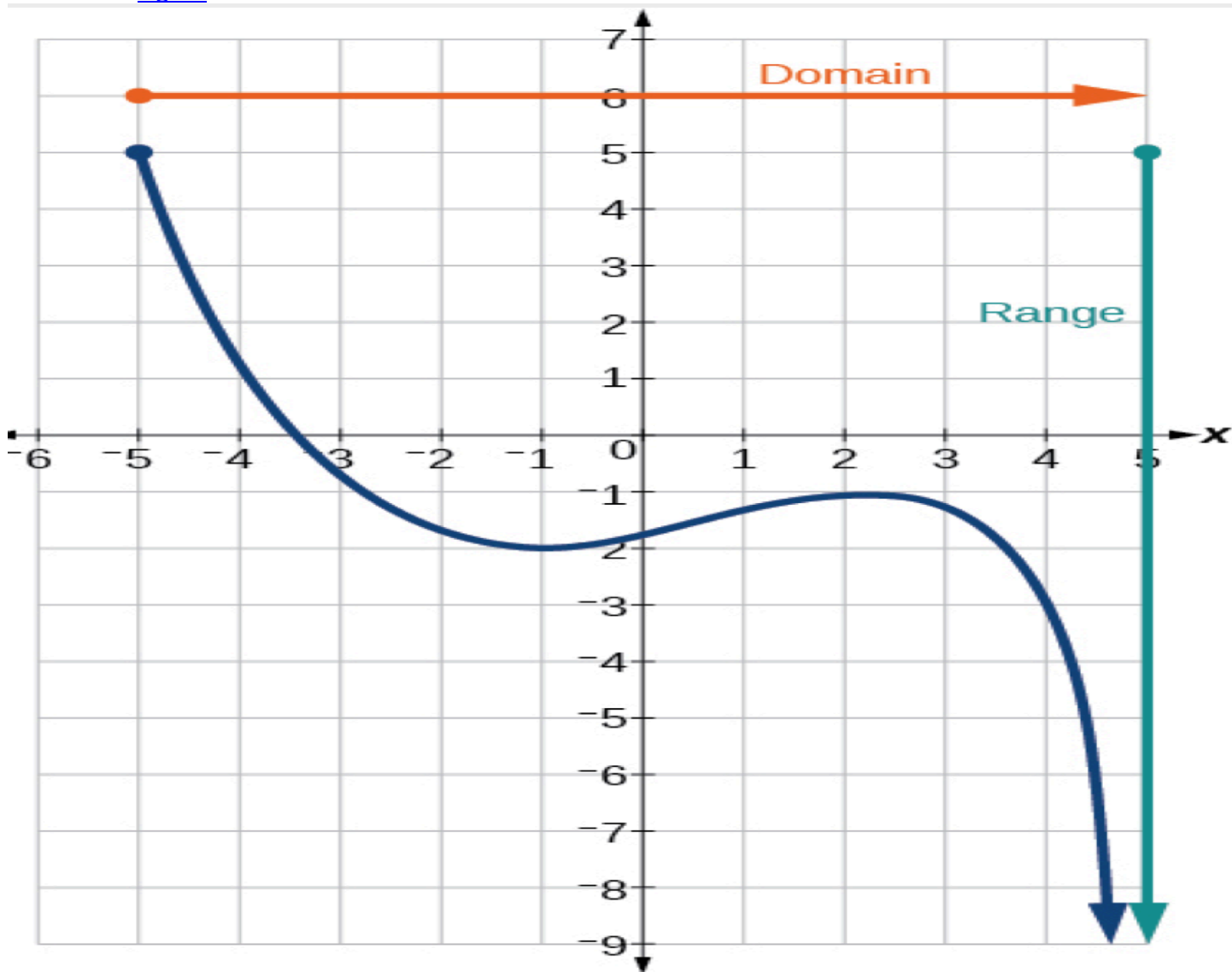
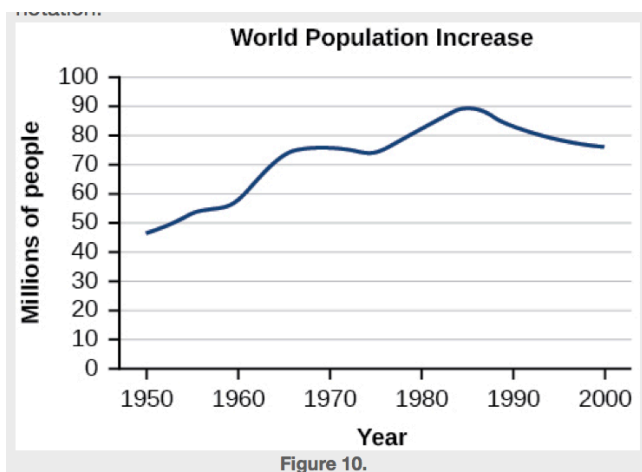
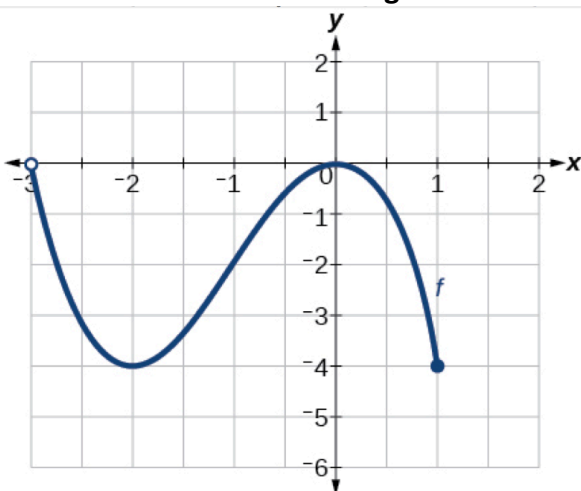


Figure 7.

Examples – Find the Domain and Range



HOW TO

Given the formula for a function, determine the domain and range.

1. Exclude from the domain any input values that result in division by zero.
2. Exclude from the domain any input values that have nonreal (or undefined) number outputs.
3. Use the valid input values to determine the range of the output values.
4. Look at the function graph and table values to confirm the actual function behavior.

Examples – check them out visually at www.desmos.com

a.

Find the domain and range of $f(x) = 2x^3 - x$.

b.

Find the domain and range of $f(x) = \frac{2}{x+1}$.

c.

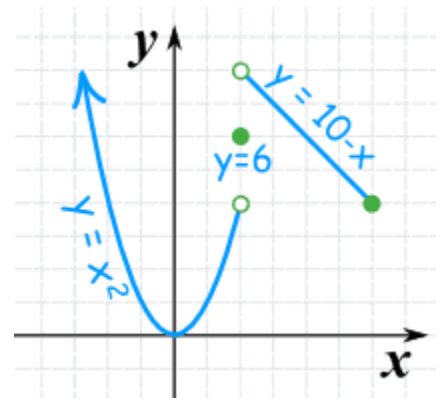
Find the domain and range of $f(x) = -\sqrt{2-x}$.

Piecewise Functions

A GENERAL NOTE: PIECEWISE FUNCTION

A **piecewise function** is a function in which more than one formula is used to define the output. Each formula has its own domain, and the domain of the function is the union of all these smaller domains. We notate this idea like this:

$$f(x) = \begin{cases} \text{formula 1} & \text{if } x \text{ is in domain 1} \\ \text{formula 2} & \text{if } x \text{ is in domain 2} \\ \text{formula 3} & \text{if } x \text{ is in domain 3} \end{cases}$$



HOW TO

Given a piecewise function, write the formula and identify the domain for each interval.

1. Identify the intervals for which different rules apply.
2. Determine formulas that describe how to calculate an output from an input in each interval.
3. Use braces and if-statements to write the function.

HOW TO

Given a piecewise function, sketch a graph.

1. Indicate on the x -axis the boundaries defined by the intervals on each piece of the domain.
2. For each piece of the domain, graph on that interval using the corresponding equation pertaining to that piece. Do not graph two functions over one interval because it would violate the criteria of a function.

Example

Writing a Piecewise Function

A museum charges \$5 per person for a guided tour with a group of 1 to 9 people or a fixed \$50 fee for a group of 10 or more people. Write a **function** relating the number of people, n , to the cost, C .

a.

b.

Graph the following piecewise function.

$$f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ -2 & \text{if } -1 < x < 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$$

