3.1 – Functions and Function Notation

A ___________________________________ is a set of ordered pairs. The set consisting of the first components of each ordered pair is called the _______________________, and is also known as the _____________ or ____ variables. The set consisting of the second components of each ordered pair is called the _________________ and is also known as the _________________ or _________________________________ variables.

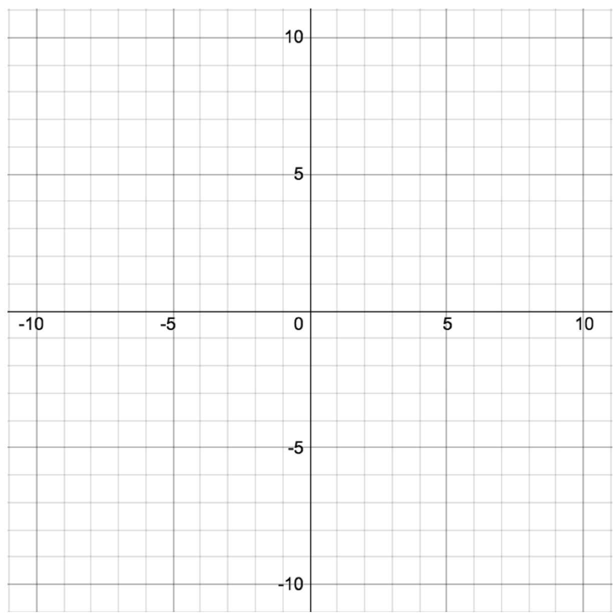
Consider the following set of ordered pairs. The first numbers in each pair are the first five natural numbers. The second number in each pair is twice that of the first.

$$\{(1,2),(2,4),(3,6),(4,8),(5,10)\}$$

Domain/Input/Independent:

Range/Output/Dependent:

x	y



A GENERAL NOTE: FUNCTION

A **function** is a relation in which each possible input value leads to exactly one output value. We say “the output is a function of the input.”

The **input** values make up the **domain**, and the **output** values make up the **range**.

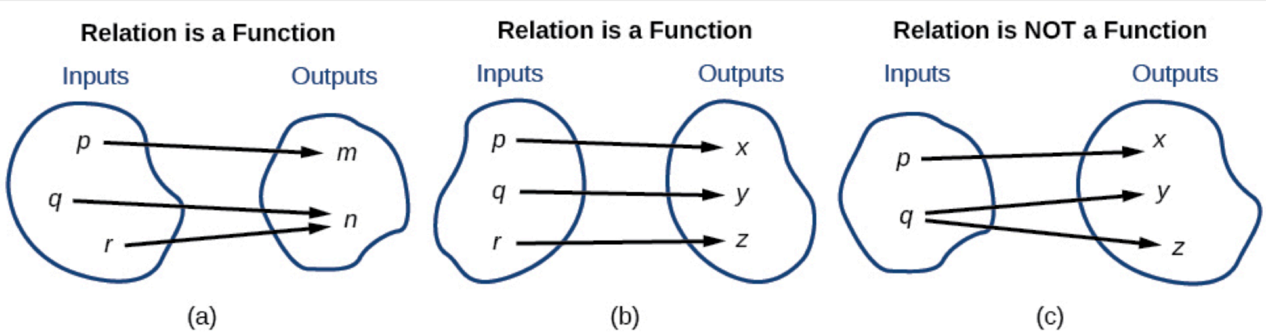


Figure 1. (a) This relationship is a function because each input is associated with a single output. Note that input q and r both give output n. (b) This relationship is also a function. In this case, each input is associated with a single output. (c) This relationship is not a function because input q is associated with two different outputs.

HOW TO

Given a relationship between two quantities, determine whether the relationship is a function.

1. Identify the input values.
2. Identify the output values.
3. If each input value leads to only one output value, classify the relationship as a function. If any input value leads to two or more outputs, do not classify the relationship as a function.

Player	Rank
Babe Ruth	1
Willie Mays	2
Ty Cobb	3
Walter Johnson	4
Hank Aaron	5

a. Is the rank a function of the player name?
b. Is the player name a function of the rank?

Determining If Class Grade Rules Are Functions

In a particular math class, the overall percent grade corresponds to a grade-point average. Is grade-point average a function of the percent grade? Is the percent grade a function of the grade-point average? Table shows a possible rule for assigning grade points.

Percent grade	0–56	57–61	62–66	67–71	72–77	78–86	87–91	92–100
Grade-point average	0.0	1.0	1.5	2.0	2.5	3.0	3.5	4.0

Using Function Notation

Once we determine that a relationship is a function, we need to display and define the functional relationships so that we can understand and use them, and sometimes also so that we can program them into graphing calculators and computers. There are various ways of representing functions. A standard - ___________________ ______________________ is one representation that facilitates working with functions.

A GENERAL NOTE: FUNCTION NOTATION

The notation $y = f(x)$ defines a function named f. This is read as "y is a function of x." The letter x represents the input value, or independent variable. The letter y, or $f(x)$, represents the output value, or dependent variable.

We can use any letter to name the function; the notation $h(a)$ shows us that h depends on a. The value a must be put into the function h to get a result. The parentheses indicate that age is input into the function; they do not indicate multiplication.

We can also give an algebraic expression as the input to a function.

For example $f(a+b)$ means "first add *a* and *b*, and the result is the input for the function *f*." The operations must be performed in this order to obtain the correct result.

Example

a. A function $N = f(y)$ gives the number of police officers, N, in a town in year y. What does $f(2005) = 300$ represent?

b. Use function notation to express the weight of a pig in pounds as a function of its age in days d.

HOW TO

Given a table of input and output values, determine whether the table represents a function.

1. Identify the input and output values.
2. Check to see if each input value is paired with only one output value. If so, the table represents a function.

Do the following tables represent functions?

Input	Output
1	10
2	100
3	1000

Input	Output
–3	5
0	1
4	5

Input	Output
1	0
5	2
5	4

Finding Input and Output Values for Functions

When we know an input value and want to determine the corresponding output value for a function, we ________________________________ the function. Evaluating will always produce one result because each input value of a function corresponds to exactly one output value.

When we know an output value and want to determine the input values that would produce that output value, we set the output equal to the function's formula and ____________________for the input. Solving can produce more than one solution because different input values can produce the same output value.

HOW TO

Given the formula for a function, evaluate.

1. Replace the input variable in the formula with the value provided.
2. Calculate the result.

Examples:

Evaluating Functions at Specific Values

Evaluate $f(x) = x^2 + 3x - 4$ at

a. 2

b. a

c. $a + h$

d. $\frac{f(a+h)-f(a)}{h}$

Evaluating Functions Expressed in Function Form

Some functions are defined by mathematical rules or procedures expressed in equation form. If it is possible to express the function output with a formula involving the input quantity, then we can define a function in algebraic form.

HOW TO

Given a function in equation form, write its algebraic formula.

1. Solve the equation to isolate the output variable on one side of the equal sign, with the other side as an expression that involves *only* the input variable.
2. Use all the usual algebraic methods for solving equations, such as adding or subtracting the same quantity to or from both sides, or multiplying or dividing both sides of the equation by the same quantity.

Examples

a. Finding an Equation of a Function

Express the relationship $2n + 6p = 12$ as a function $p = f(n)$, if possible.

b. Does the equation $x^2 + y^2 = 1$ represent a function with x as input and y as output? If so, express the relationship as a function $y = f(x)$.

c. If $x - 8y^3 = 0$, express y as a function of x.

Evaluating a Function Given in Tabular Form

HOW TO

Given a function represented by a table, identify specific output and input values.

1. Find the given input in the row (or column) of input values.
2. Identify the corresponding output value paired with that input value.
3. Find the given output values in the row (or column) of output values, noting every time that output value appears.
4. Identify the input value(s) corresponding to the given output value.

Examples

Using Table,

a. Evaluate $g(3)$.
b. Solve $g(n) = 6.$

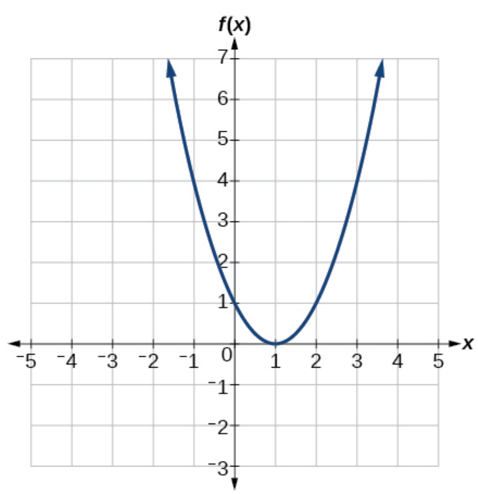
n	1	2	3	4	5
$g(n)$	8	6	7	6	8

Finding Function Values from a Graph

Evaluating a function using a graph also requires finding the corresponding output value for a given input value, only in this case, we find the output value by looking at the graph. Solving a function equation using a graph requires finding all instances of the given output value on the graph and observing the corresponding input value(s)

Examples

a. **Find *f(-1)***
b. **Solve *f(x) = 1***



Determining Whether a Function is One-To-One

Some functions have a given output value that corresponds to two or more input values. However, some functions have only one input value for each output value, as well as having only one output for each input. We call these functions ________-_____-________ functions

Example

a. If each percent grade earned in a course translates to one letter grade, is the letter grade a function of the percent grade?
b. If so, is the function one-to-one?

Using The Vertical Line Test

The ______________________ ___________ _____________ can be used to determine whether a graph represents a function. If we can draw any vertical line that intersects a graph more than once, then the graph does *not* define a function because a function has only one output value for each input value. If the function sends any input to more than one output, the resulting graph will have points that stack on top of each other, resulting in a failure of the vertical line test for a function.

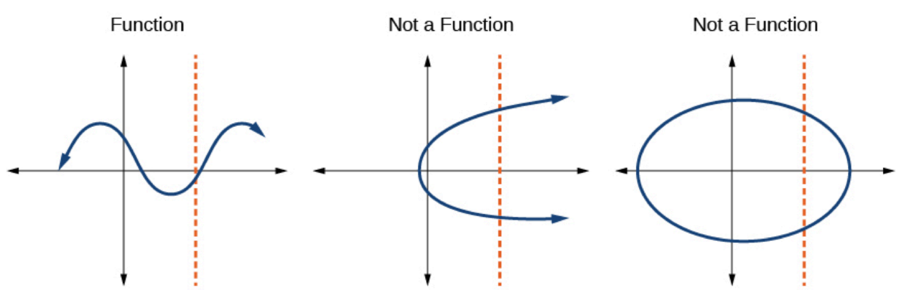


Figure 5.

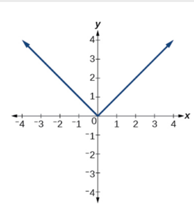
HOW TO

Given a graph, use the vertical line test to determine if the graph represents a function.

1. Inspect the graph to see if any vertical line drawn would intersect the curve more than once.
2. If there is any such line, determine that the graph does not represent a function.

Example:

a. **Does the graph represent a function?**
b. **Is the function one-to-one?**



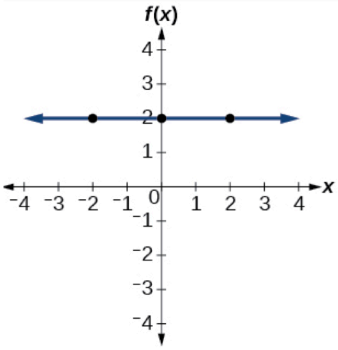
HOW TO

Given a graph of a function, use the horizontal line test to determine if the graph represents a one-to-one function.

1. Inspect the graph to see if any horizontal line drawn would intersect the curve more than once.
2. If there is any such line, determine that the function is not one-to-one.

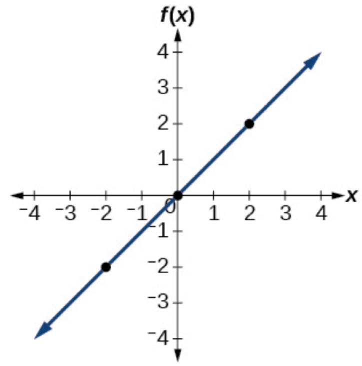
Identifying Basic Toolkit Functions (Parent Functions)

Constant $f(x) = c,$ where c is a constant



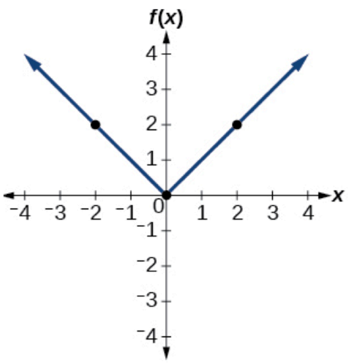
x	f(x)
−2	2
0	2
2	2

Identity $f(x) = x$



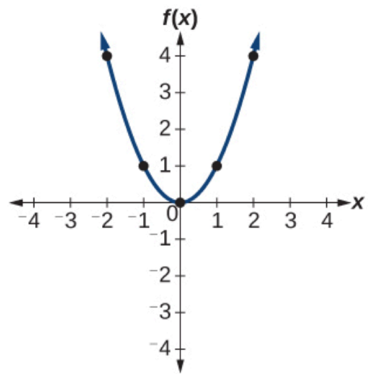
x	f(x)
−2	−2
0	0
2	2

Absolute value $f(x) = |x|$



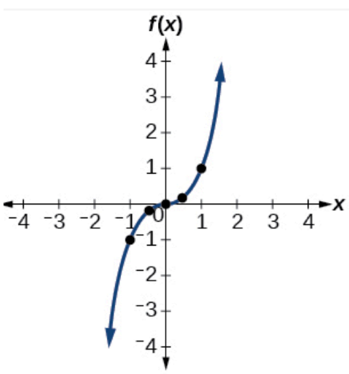
x	f(x)
−2	2
0	0
2	2

Quadratic $f(x) = x^2$



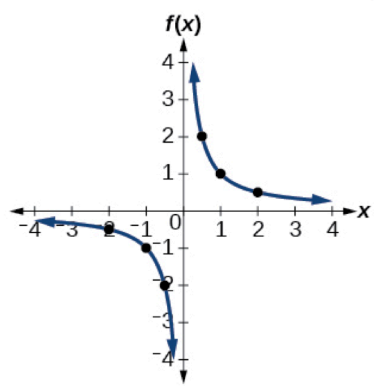
x	f(x)
−2	4
−1	1
0	0
1	1
2	4

Cubic $f(x) = x^3$



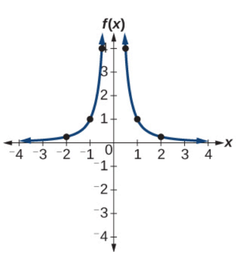
x	f(x)
−1	−1
−0.5	−0.125
0	0
0.5	0.125
1	1

Reciprocal $f(x) = \frac{1}{x}$



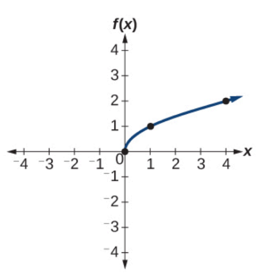
x	f(x)
−2	−0.5
−1	−1
−0.5	−2
0.5	2
1	1
2	0.5

Reciprocal squared $f(x) = \frac{1}{x^2}$



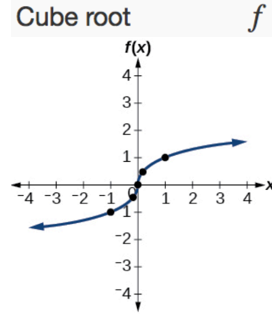
x	f(x)
−2	0.25
−1	1
−0.5	4
0.5	4
1	1
2	0.25

Square root $f(x) = \sqrt{x}$



x	f(x)
0	0
1	1
4	2

Cube root $f(x) = \sqrt[3]{x}$



x	f(x)
−1	−1
−0.125	−0.5
0	0
0.125	0.5
1	1