### Hyperbolas KEY

### Objectives

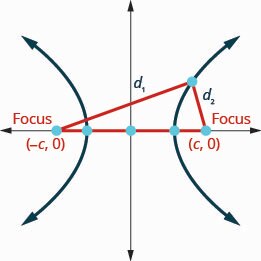
* Graph a hyperbola with center at
* Identify conic sections by their equations

### Graph a Hyperbola with Center at (0, 0)

The last conic section we will look at is called a hyperbola. We will see that the equation of a hyperbola looks similar to the equation of an **ellipse**, except it is a **difference** rather than a sum.

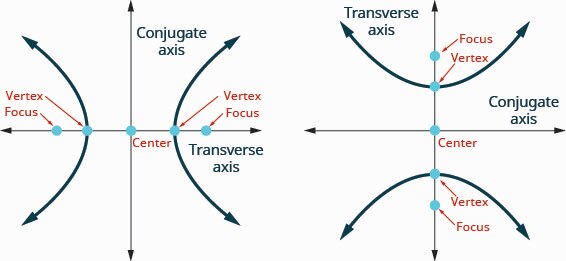
Hyperbola

A **hyperbola** is all points in a plane where the **difference** of their distances from two fixed points is **constant**. Each of the fixed points is called a **focus** of the hyperbola.



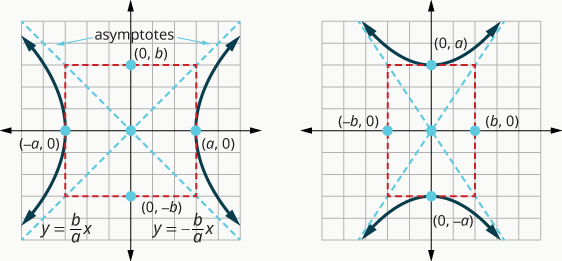
Definitions:

* The line through the foci is called the **transverse axis**.
* The two points where the transverse axis intersects the hyperbola are each a **vertex** of the hyperbola.
* The midpoint of the segment joining the foci is called the **center** of the hyperbola.
* The line perpendicular to the transverse axis that passes through the center is called the **conjugate axis**.
* Each piece of the graph is called a **branch** of the hyperbola.



Standard Form of the Equation a Hyperbola with Center

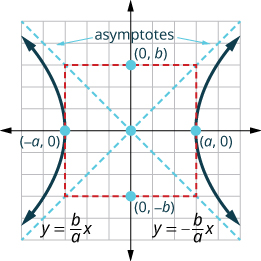
The standard form of the equation of a hyperbola with center is given two ways:



* **A** is the distance from the center to **vertices** along the **transverse** axis.
* **B** is the distance from the center to the points on the **conjugate** axis, used to determine how **wide** the hyperbola opens.
* **C** is the distance from the center to the **foci**.
* Pythagorean relationship: **.**
* The **transverse axis** contains the vertices and foci and is determined by which term is **positive**.

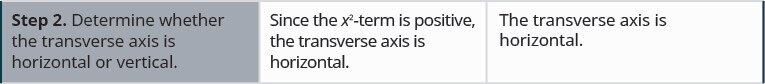
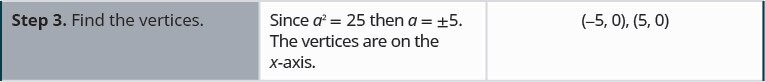
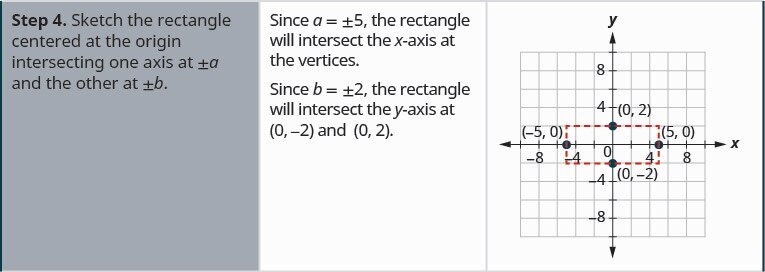
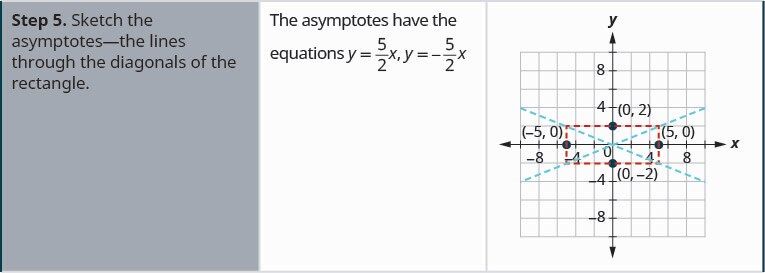
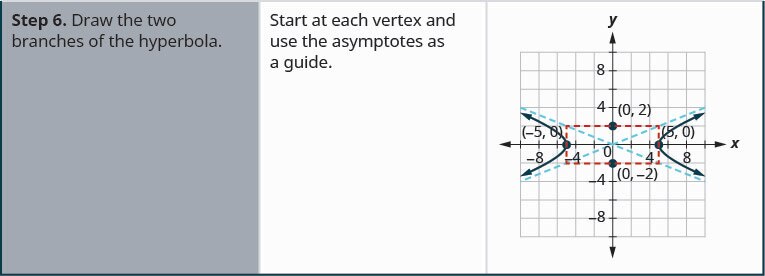
**Asymptotes**

The **asymptotes** are intersecting straight lines that the branches of the graph **approach** but never intersect as the *x*, *y* values get larger and larger. The asymptotes are determined by the values of ***a* and *b***.



To find the asymptotes, we sketch a **rectangle** whose sides are based off the values of *a* and *b*. The lines containing the **diagonals** of this rectangle are the asymptotes of the hyperbola. The rectangle and asymptotes are not part of the hyperbola, but they help us sketch the graph.

1. Graph the hyperbola.

Step 1 is to write the equation in standard form. The the quantity x squared divided by 25 end quantity minus the quantity y squared divided by 4 end quantity is equal to 1 is already in standard form.          

1. Graph the hyperbola.

|  |  |
| --- | --- |
| **To write the equation in standard form, divide each term by 64 to make the equation equal to 1.** |  |
| **Simplify.** |  |
| **Since the *y*2-term is positive, the transverse axis is vertical. Since then** |  |
| **The vertices are on the *y*-axis, Since then** | **The vertices are** |
| **The foci are on the y-axis, (0, -c), (0, c). Since , we have that =20. Therefore, .** | **The foci are .** |
| **Sketch the rectangle intersecting the *x*-axis at and the *y*-axis at the vertices. Sketch the asymptotes through the diagonals of the rectangle. Draw the two branches of the hyperbola.** | **.** |

1. Find an equation of a hyperbola centered at the origin with a vertex at (0,3) and a focus at (0,-6). Then sketch the graph.

**The transverse axis is the y-axis, so the equation will be of the form . From what is given, we have that and . To find , use the Pythagorean relationship. We know that , meaning that and so . Therefore, the equation is given by .**

Chart, line chart

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### Identify Conic Sections by their Equations

To identify a conic from its equation, put the **variable** terms on one side of the equation and the **constants** on the other. It may be helpful to put the equations in **standard form**.

|  |  |  |
| --- | --- | --- |
|  |  | Example |
| **Parabola** | Either OR Only **one** variable is squared. |  |
| **Circle** | and terms have the **same** **coefficients** |  |
| **Ellipse** | and terms have the **same** **sign**, different coefficients |  |
| **Hyperbola** | and terms have **different signs**, different coefficients |  |

Identify the graph of each equation as a circle, parabola, ellipse, or hyperbola. Then put the equation in standard form

|  |  |
| --- | --- |
|  |  |
| **The - and -terms have the same sign and different coefficients.** | **Ellipse** |
| **To put in standard form, divide everything by 72.** |  |
| **Simplify.** |  |



|  |  |
| --- | --- |
|  |  |
| **The - and -terms have the same coefficients.** | **Circle** |
| **To put in standard form, complete the square.** |  |



|  |  |
| --- | --- |
|  |  |
| Only one variable, , is squared. | Parabola |
| To put in standard form, complete the square. |  |

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