6.6 Graphing: Putting it all Together

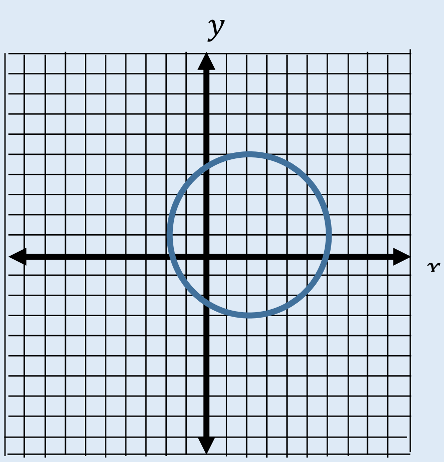
At this point in the course, you should know quite a bit about graphing. We have learned about thirteen different relations and methods for graphing them. We have learned about domain, range, intercepts, asymptotes, functions, and inverses. The goal of this section is to help you identify the type of graph in a given problem in order to proceed with the appropriate method for graphing. This will be a very important skill to have for your future math courses, especially calculus. You will gather more graphs to add to this repertoire in Trigonometry and other courses. You will use many of the same skills and methods you have learned in this course. A graphing guide is provided at the end of this section with some helpful ways to organize the information for retrieval!

Examples

For each of the following relations, identify the type of graph and then graph it using the appropriate method. Give information about the graph such as domain, range, intercepts, and asymptotes. Also, state whether or not the relation is a function and one-to-one.

1. $(x-2)^2 + (y-1)^2 = 16$

This is a circle of radius 4 centered at $(2, 1)$.



Domain: $[-2, 6]$ Range: $[-3, 5]$

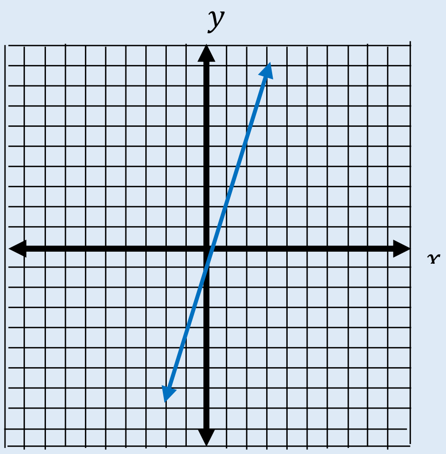
x-int: $(2 \pm \sqrt{15}, 0)$ y-int: $(0, 1 \pm 2\sqrt{3})$

Not a function (vertical line test)

Not one-to-one (since not a function)

2. $y = 3x - 1$

This is a line with slope 3 and y-int $(0, -1)$.



Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

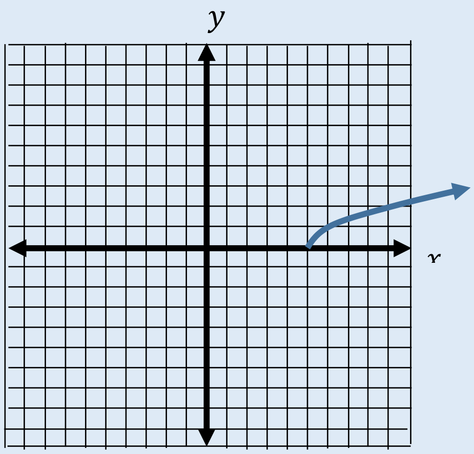
x-int: $(\frac{1}{3}, 0)$ y-int: $(0, -1)$

It is a function (vertical line test)

It is one-to-one (horizontal line test)

3. $y = \sqrt{x - 5}$

This is a basic radical function shifted 5 to the right.



Domain: $[5, \infty)$ Range: $[0, \infty)$

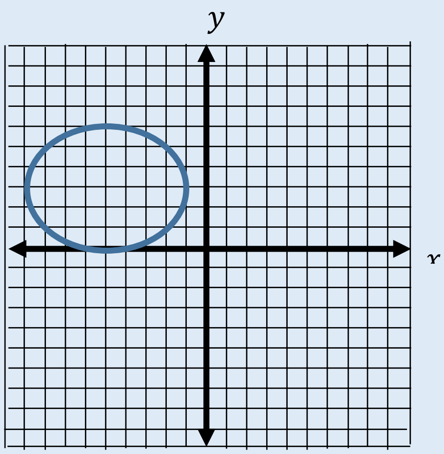
x-int: $(5, 0)$ y-int: none

It is a function (vertical line test)

It is one-to-one (horizontal line test)

4. $\frac{(x+5)^2}{16}+\frac{(y-3)^2}{9}=1$

This is an ellipse centered at $(-5,3)$ with $a=4$ and $b=3$.



Domain: $[-9,-1]$ Range: $[0,6]$

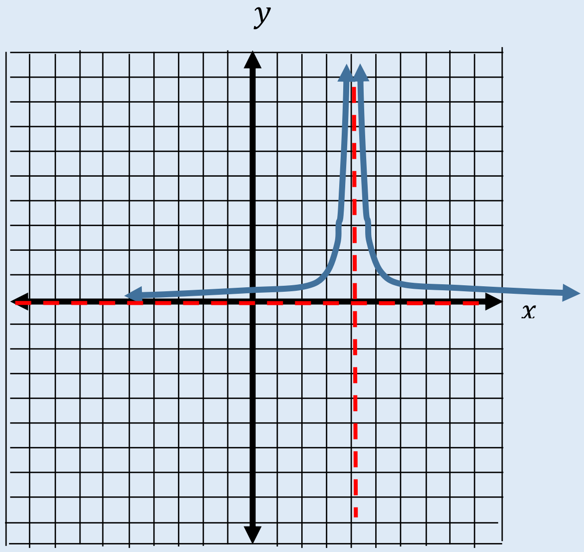
x-int: $(-3,0)$ y-int: none

Not a function (vertical line test)

Not one-to-one (since not a function)

5. $f(x)=\frac{1}{(x-4)^2}$

This is a basic rational square graph (positive ramps) shifted to the right by 4.



Domain: $(-\infty,4)\cup(4,\infty)$

Range: $(0,\infty)$

x-int: none y-int: none

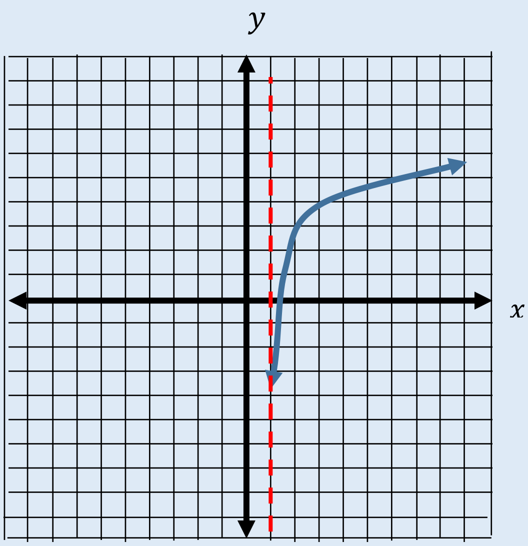
It is a function (vertical line test)

It is not one-to-one (horizontal line test)

Asymptotes: $x=4; y=0$

6. $g(x) = log_2(x-1) + 3$

This is a basic logarithmic graph shifted 1 to the right and up 3.



Domain: $(1, \infty)$ Range: $(-\infty, \infty)$

x-int: $\left(\frac{9}{8}, 0\right)$ y-int: none

It is a function (vertical line test)

It is one-to-one (horizontal line test)

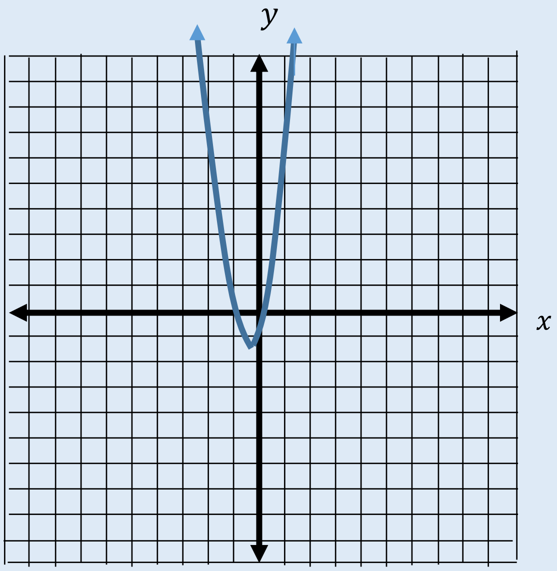
Asymptote: $x = 1$

7. $y - 4x^2 = 2x - 1$

This is a parabola since one of the variables is squared. Completing the square to put it in graphing form:

$$y = 4x^2 + 2x - 1$$
$$y = 4\left(x^2 + \frac{1}{2}x - \frac{1}{4}\right)$$
$$y = 4\left(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} - \frac{1}{4}\right)$$
$$y = 4\left(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} - \frac{1}{4}\right)$$
$$y = 4\left[\left(x + \frac{1}{4}\right)^2 - \frac{5}{16}\right]$$
$$y = 4\left(x + \frac{1}{4}\right)^2 - \frac{5}{4}$$

This parabola opens up and has a vertex of $\left(-\frac{1}{4}, -\frac{5}{4}\right)$.



Domain: $(-\infty, \infty)$ Range: $[-\frac{5}{4}, \infty)$

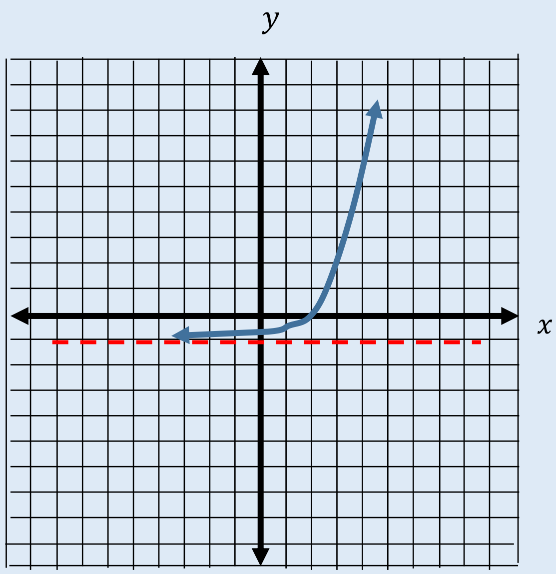
x-int: $(\frac{-1\pm\sqrt{5}}{4}, 0)$ y-int: $(0, -1)$

It is a function (vertical line test)

It is not one-to-one (horizontal line test)

8. $y = 3^{x-2} - 1$

This is a basic exponential function shifted to the right by 2 and down by 1.



Domain: $(-\infty, \infty)$ Range: $(-1, \infty)$

x-int: $(2, 0)$ y-int: $\left(0, -\frac{8}{9}\right)$

It is a function (vertical line test)

It is one-to-one (horizontal line test)

Asymptote: $y = -1$

General approach to graphing

1. Is it linear? (i.e. a line) You know that it is if both x and y are to the first power. If so, you can either plug in two points and connect them or you can put it in slope-intercept form (y=mx+b) and use the y-intercept and slope to get two points and connect them.

2. If it is not linear, then is it one of the basic graphs that can be shifted/refelected? These are:

   a. $y = x^2$ (basic parabola - bowl)
   b. $y = x^3$ (basic cubic - snake)
   c. $y = |x|$ (basic absolute value - V)
   d. $y = \sqrt{x}$ (basic radical – half of sideways parabola)
   e. $y = \frac{1}{x}$ (basic rational – positive and negative ramps)
   f. $y = \frac{1}{x^2}$ (basic rational – positive ramps)
   g. $y = b^x$ (basic exponential – single ramp)
   h. $y = log_b x$ (basic logarithmic – upside down ramp)

If so, graph the basic graph by plotting a few points that will give you the shape/curvature and then shift/reflect as indicated in the equation.

3. If it is not linear and it is not one of the basic graphs above, then is it a conic? If both x and y are squared, then that would indicate it is a conic and you may need to put it in graphing form by completing the square. You might also have only one of the variables being squared and the other is to the first power (then it is a parabola):

   a. Parabola: $y = a(x-h)^2 + k \ \ or\ x = a(y-k)^2 + h$
   b. Circle: $(x-h)^2 + (y-k)^2 = r^2$
   c. Ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
   d. Hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \ or\ \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$