4.6 Solving Inequalities – Putting it all together

Why do we solve inequalities? We solve inequalities for many of the same reasons that we solve equations. Solving inequalities allows us to solve problems that have a range of conditions and solutions. This is often the case in business, medicine, and engineering, where a solution is dependent on a minimum or maximum value (or a range) of a particular quantity. Any time we encounter a situation where we have "at least", "at most", or a range of values to stay within, we know that an inequality will be involved. Inequalities also help when it comes to graphing polynomial and rational functions by telling us where the graph is (above or below the x-axis) for each interval. In a similar way, they allow more complex analysis of functions representing quantities in order to determine maximum and minimum values for those quantities (e.g. maximizing profit and minimizing loss in business, maximizing or minimizing surface area in engineering, minimizing effective dosage in medicine, etc.). This analysis is done in a calculus course, but it is relatively easy to do if you have the algebraic background you need!

In this section, we will practice identifying the technique required to solve a given inequality. You need to be able to recognize the type of inequality you are dealing with and then identify the technique that will help you solve it. We only have four types to distinguish between in this course: linear, absolute value linear, compound, and polynomial/rational.

We will do a couple examples to help demonstrate the thought process. It is important to practice recognizing the type of inequality because that can serve as a recall "cue" to help us remember the technique (provided we have enough practice with each technique).

Examples

Solve each of the following inequalities for the indicated variable.

1. $-3|x+4|+10 \le -2$

We should recognize the absolute value symbols in this inequality, which means we need to isolate the absolute value and then split into two inequalities (as long as the absolute value isn't less than a negative number, since then there would be no solution).

$$\begin{array}{rr} -3|x+4|+10 & \le -2 \\ -10 & -10 \\ \hline \end{array}$$

⇨ $-3|x+4| \le -12$

Don't forget to reverse the inequality when dividing by a negative number.

⇨ $\dfrac{-3|x+4|}{-3} \ge \dfrac{-12}{-3}$

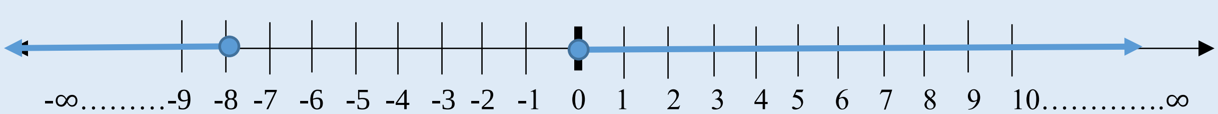
⇨ $|x+4| \ge 4$

$$\begin{array}{rr} x+4 & \ge 4 \\ -4 & -4 \\ \hline \end{array} \qquad \text{or} \qquad \begin{array}{rr} x+4 & \le -4 \\ -4 & -4 \\ \hline \end{array}$$

⇨ $x \ge 0$ ⇨ $x \le -8$

So we have the solution " $x \ge 0$ or $x \le -8$ ".

The graph is as follows:



The solution is $(-\infty, -8] \cup [0, \infty)$.

2. $2x^2 + 11x > 21$

We should recognize the quadratic in this inequality, which means this is a polynomial inequality and will require a sign chart.

$$2x^2 + 11x - 21 > 0$$

$$(2x - 3)(x + 7) > 0$$

Critical values: $\frac{3}{2}$ $and - 7$

The test intervals are $(-\infty, -7), \left(-7, \frac{3}{2}\right), \left(\frac{3}{2}, \infty\right)$

Test value:	-8	0	2
	$(-\infty, -7)$	$(-7, \frac{3}{2})$	$\left(\frac{3}{2}, \infty\right)$
$2x - 3$	$-$	$-$	$+$
$x + 7$	$-$	$+$	$+$
Result:	$+$	$-$	$+$

The solution is $(-\infty, 5) \cup \left(\frac{3}{2}, \infty\right)$.