4.1 Solving Equations – Putting it all together

Why do we solve equations? Solving equations allows us to solve problems in many different fields of study as well as in our own lives. Equations model real situations that require solutions. Quadratics and other polynomial equations can represent descriptors of motion in physics, such as position, velocity and acceleration. They can also represent the size of objects, such as surface area and volume in engineering applications. Exponential and logarithmic equations are needed to model growth and decay processes in biology, environmental science, geology and even finance. Linear equations are used to model supply and demand in economics and correlation in statistics. These are just a few examples of where you see equations in the world around you.

In this section, we will practice identifying the technique required to solve a given equation. You need to be able to recognize the type of equation you are dealing with and then identify the technique that will help you solve it. This is a very important skill and it will help you throughout the rest of your math and science courses and possibly in your career as well!

Another thing we should know about equations is when we need to check our answers. You can always check them if you want to make sure you didn't make any errors, but you *need* to check them for any equations that have variables inside of denominators, radicals, or logarithms. We know that sometimes we end up with extraneous solutions in these cases. Polynomial equations will not have extraneous solutions, although sometimes they may have imaginary or complex solutions. Those are allowed, however.

<u>Examples</u>

Solve each of the following equations for the indicated variable.

1. $3|-2x+7|-13=0$

We should recognize the absolute value symbols in this equation, which means we need to isolate the absolute value and then split into two equations (as long as it doesn't equal a negative number, since then there would be no solution).

$$\begin{array}{r} 3|-2x+7|-13=-7 \\ +13 \quad +13 \\ \hline \end{array}$$

$\Rightarrow \quad 3|-2x+7|=6$

$\Rightarrow \quad \dfrac{3|-2x+7|}{3}=\dfrac{6}{3}$

$\Rightarrow \quad |-2x+7|=2$

Once you isolate the absolute value, don't forget to check whether or not this is solvable. Since the absolute value is equal to a positive number, we can proceed.

$$\begin{array}{r} -2x+7=\ 2 \\ -7 \quad -7 \\ \hline -2x \quad =-5 \end{array} \qquad \text{or} \qquad \begin{array}{r} -2x+7=-2 \\ -7 \quad -7 \\ \hline -2x \quad =-9 \end{array}$$

$\Rightarrow \quad x=\frac{5}{2};\ \ x=\frac{9}{2}$

2. $5x^3 + 10x^2 - 12x = 24$

We should be able to identify this one quickly as a polynomial equation and recall that we should move everything to one side and factor…..

$$5x^3 + 10x^2 - 12x = 24$$

Since there are four terms in the polynomial, we need to try grouping to factor.

$$\Rightarrow \quad 5x^3 + 10x^2 - 12x - 24 = 0$$

$$\Rightarrow \quad 5x^2(x + 2) - 12(x + 2) = 0$$

$$\Rightarrow \quad (x + 2)(5x^2 - 12) = 0$$

$$\Rightarrow \quad x + 2 = 0 \quad ; \quad 5x^2 - 12 = 0$$

$$x = -2 \qquad\qquad 5x^2 = 12$$

$$x^2 = \frac{12}{5}$$

$$\sqrt{x^2} = \pm\sqrt{\frac{12}{5}}$$

$$x = \pm\frac{2\sqrt{3}}{\sqrt{5}} = \pm\frac{2\sqrt{15}}{5}$$

Don't forget to rationalize the denominators in your answers when applicable.

$$\Rightarrow \quad x = -2, \frac{2\sqrt{15}}{5}, -\frac{2\sqrt{15}}{5}$$

3. $\sqrt{2y-1} - \sqrt{y+3} = 1$

Here we have an equation involving radicals, so we should isolate one of the radicals and square both sides. We will have to repeat the process here since we have two radicals.

$$\sqrt{2y-1} - \sqrt{y+3} = 1$$

$$\Rightarrow \quad \sqrt{2y-1} - \sqrt{y+3} = 1$$

$$\qquad\qquad +\sqrt{y+3} \quad +\sqrt{y+3}$$

$$\Rightarrow \quad \sqrt{2y-1} = 1 + \sqrt{y+3}$$

$$\Rightarrow \quad \left(\sqrt{2y-1}\right)^2 = \left(1 + \sqrt{y+3}\right)^2$$

$$\Rightarrow \quad 2y - 1 = 1 + 2\sqrt{y+3} + y + 3$$

$$\Rightarrow \quad 2y - 1 \ = 4 + 2\sqrt{y+3} + y$$

$$-y - 4 \quad -4 \qquad\qquad -y$$

$$\Rightarrow \quad y - 5 \ = 2\sqrt{y+3}$$

$$\Rightarrow \quad (y - 5\,)^2 = \left(2\sqrt{y+3}\right)^2$$

$$\Rightarrow \quad y^2 - 10y + 25 \ = 4(y+3)$$

$$\Rightarrow \quad y^2 - 10y + 25 \ = 4y + 12$$

$$\qquad -4y - 12 \quad -4y - 12$$

$$\Rightarrow \quad y^2 - 14y + 13 \ = 0$$

Now we have a quadratic, so bring everything to one side and factor or use the quadratic formula. We are used to using multiple techniques in one problem by now! ☺

➪ $(y-13)(y-1) \ = 0$

➪ $y = 13; \quad y = 1$

Don't forget we have to check our answers because our original equation had radicals:

$y = 13$	$y = 1$
$\sqrt{2 \cdot 13 - 1} - \sqrt{13+3} = 1$	$\sqrt{2 \cdot 1 - 1} - \sqrt{1+3} = 1$
$\sqrt{25} - \sqrt{16} = 1$	$\sqrt{1} - \sqrt{4} = 1$
$5 - 4 = 1$ ✓	$1 - 2 = 1$ Not true

So we have one answer: $y = 13$

4. $log_6(x+1) + log_6(x-4) = 2$

We need to use the rules of logarithms to combine these logarithms and then use the definition to move the base to the other side of the equation (rewrite it as an exponential).

➪ $log_6(x+1)(x-4) = 2$

➪ $(x+1)(x-4) = 6^2$

➪ $x^2 - 3x - 4 = 36$

➪ $x^2 - 3x - 40 = 0$

➪ $(x-8)(x+5) = 0$

➪ $x = 8; \quad x = -5$

We must check that our answers do not make the arguments negative since we know the logarithm of a negative number is undefined:

The answer $x = 8$ is fine since $8 + 1$ and $8 - 4$ are both positive. The answer $x = -5$ is extraneous since $-5 + 1$ is negative.

So we have one answer: $x = 8$

5. $9^{5t} = 27^{t^2+1}$

This is an exponential equation and we can tell because the variables occur in the exponents. First, we will check to see if we can write both sides with the same base since that is easier and if we can't, then we will take the same logarithm of both sides.

$9^{5t} = 27^{t^2+1}$

➪ $(3^2)^{5t} = (3^3)^{t^2+1}$

➪ $3^{10t} = 3^{3t^2+3}$

➪ $10t = 3t^2 + 3$

➪ $0 = 3t^2 - 10t + 3$

Since 9 and 27 can both be written as a power of 3, we are in luck!

Using the rule $(b^m)^n = b^{mn}$, don't forget to distribute the 3 to both t^2 and 1.

Since the bases are equal, we can set the exponents equal to each other.

⇨ $0 = (3t-1)(t-3)$

⇨ $t = \frac{1}{3};\ \ t = 3$

6. $2e^{2x+4} - 3e^{x+2} = -1$

This one involves more thought. We know that it is exponential, but there is more than one variable exponent here. Since it looks complicated, we should try a substitution. Is the exponent on the first term's exponential double the exponent on the second term's exponential? Yes, because $2x + 4 = 2(x+2)$.

This equation could be written:

$$2(e^{x+2})^2 - 3e^{x+2} = -1$$

$$\text{Let } y = e^{x+2}$$

Then we can rewrite our equation as:

$$2y^2 - 3y = -1$$

Much better, right?

⇨ $2y^2 - 3y + 1 = 0$

⇨ $(2y-1)(y-1) = 0$

⇨ $y = \frac{1}{2};\ \ \ y = 1$

Now, we need to find our original variable x by replacing y with e^{x+2} in our solutions:

$$\Rightarrow \qquad e^{x+2} = \frac{1}{2}; \quad e^{x+2} = 1$$

Solve each of these for x by taking the natural logarithm of both sides:

$$\Rightarrow \qquad \ln(e^{x+2}) = \ln\left(\frac{1}{2}\right); \quad \ln(e^{x+2}) = \ln(1)$$

$$\Rightarrow \qquad x + 2 = -\ln(2) \qquad \mathrm{x} + 2 = 0$$

$$\Rightarrow \qquad x = -2 - \ln(2) \qquad \mathrm{x} = -2$$