1.10 Simplifying Complex Fractions

Complex fractions are just fractions within fractions, so there may be a fraction in either the numerator or the denominator or both. When this happens, one way to approach the problem is to get the numerator as a single fraction and get the denominator as a single fraction so that you can flip the denominator and multiply. It is not the only way, but we find it to be the simplest in most cases. (Another method would be to find the LCD for all terms in the numerator and denominator and then multiply both of them by this to clear the fractions, but we find that students often forget at least one term, so we refrain from using that method and instead focus on this method, which also reinforces the operations we covered in recent lessons). Once again, we will begin with arithmetic and apply the same procedure in algebra.

Arithmetic Example:

$$\frac{\frac{1}{3}+\frac{1}{2}}{\frac{1}{5}-\frac{1}{4}} = \frac{\frac{2}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{3}{3}}{\frac{4}{4}\cdot\frac{1}{5}-\frac{1}{4}\cdot\frac{5}{5}} = \frac{\frac{2}{6}+\frac{3}{6}}{\frac{4}{20}-\frac{5}{20}} = \frac{\frac{5}{6}}{\frac{-1}{20}} = \frac{5}{6} \div \frac{-1}{20}$$

$$= \frac{5}{6}\cdot\frac{-20}{1} = \frac{5}{6}\cdot\frac{-5\cdot\cancel{2}\cdot 2\cdot 5}{\cancel{2}\cdot 3} = -\frac{50}{3}$$

Rewrite the numerator as one fraction and the denominator as one fraction. To do this, get a common denominator for the numerator and the denominator separately and simplify. Then rewrite the fraction as division (numerator divided by denominator.)

We will now apply the same process on our algebraic examples.

Examples:

1. $$\frac{\frac{x}{2}+\frac{2x}{3}}{\frac{1}{x}-\frac{x}{2}} = \frac{\frac{3}{3}\cdot\frac{x}{2}+\frac{2x}{3}\cdot\frac{2}{2}}{\frac{2}{2}\cdot\frac{1}{x}-\frac{x}{2}\cdot\frac{x}{x}}$$

Get common denominator in numerator and denominator, then simplify. Numerator LCD= 6; Denominator LCD= $2x$

$$= \frac{\frac{3x+4x}{6}}{\frac{2-x^2}{2x}}$$

$$= \frac{\frac{7x}{6}}{\frac{2-x^2}{2x}}$$

Simplify numerator and denominator.

$$= \frac{7x}{6} \div \frac{2-x^2}{2x}$$

Rewrite as numerator ÷ denominator.

$$= \frac{7x}{6} \cdot \frac{2x}{2-x^2}$$

Rewrite as multiplication by inverting $\frac{2-x^2}{2x}$.

$$= \frac{7x}{\cancel{2}\cdot 3} \cdot \frac{\cancel{2}x}{2-x^2}$$

Simplify by factoring and cancelling, if possible.

$$= \frac{7x^2}{3(2-x^2)}$$

2. $$\frac{1+\frac{1}{x}}{1-\frac{1}{x^2}} = \frac{\frac{x}{x}\cdot\frac{1}{1}+\frac{1}{x}}{\frac{x^2}{x^2}\cdot\frac{1}{1}-\frac{1}{x^2}}$$

Get common denominator in numerator and denominator, then simplify. Numerator LCD= x; Denominator LCD= x^2

$$= \frac{\frac{x}{x}+\frac{1}{x}}{\frac{x^2}{x^2}-\frac{1}{x^2}}$$

Simplify numerator and denominator.

$$= \frac{\frac{x+1}{x}}{\frac{x^2-1}{x^2}}$$

Rewrite as numerator ÷ denominator.

$$= \frac{x+1}{x} \div \frac{x^2-1}{x^2}$$

Rewrite as multiplication by inverting $\frac{2-x^2}{2x}$.

$$= \frac{x+1}{x} \cdot \frac{x^2}{x^2-1}$$

Rewrite as multiplication by inverting $\frac{2-x^2}{2x}$.

$$= \frac{(\cancel{x+1})\cdot\cancel{x}\cdot x}{\cancel{x}(\cancel{x+1})(x-1)}$$

Simplify by factoring and cancelling, if possible.

$$= \frac{x}{x-1}$$

3. $$\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}-\frac{1}{y}} = \frac{\frac{y^2}{y^2}\cdot\frac{1}{x^2} - \frac{1}{y^2}\cdot\frac{x^2}{x^2}}{\frac{y}{y}\cdot\frac{1}{x} - \frac{1}{y}\cdot\frac{x}{x}}$$

Get common denominator in numerator and denominator, then simplify. Numerator LCD= x^2y^2; Denominator LCD= xy

$$= \frac{\frac{y^2}{x^2y^2} - \frac{x^2}{x^2y^2}}{\frac{y-x}{xy}}$$

Simplify numerator and denominator.

$$= \frac{\frac{y^2-x^2}{x^2y^2}}{\frac{y-x}{xy}}$$

Rewrite as numerator ÷ denominator.

$$= \frac{y^2-x^2}{x^2y^2} \div \frac{y-x}{xy}$$

Rewrite as multiplication by inverting $\frac{y-x}{xy}$.

$$= \frac{y^2-x^2}{x^2y^2} \cdot \frac{xy}{y-x}$$

Simplify by factoring and cancelling, if possible.

$$= \frac{(y+x)\cancel{(y-x)}}{\cancel{x}\cdot x\cdot\cancel{y}\cdot y} \cdot \frac{\cancel{xy}}{\cancel{y-x}}$$

$$= \frac{y+x}{xy}$$

4. $$\frac{\frac{4}{t}}{4+\frac{1}{t}} = \frac{\frac{4}{t}}{\frac{t}{t}\cdot\frac{4}{1}+\frac{1}{t}}$$

$$= \frac{\frac{4}{t}}{\frac{4t+1}{t}}$$

$$= \frac{4}{t} \div \frac{4t+1}{t}$$

$$= \frac{4}{\cancel{t}} \cdot \frac{\cancel{t}}{4t+1}$$

$$= \frac{4}{4t+1}$$

5. $$\frac{\frac{1}{r}-\frac{1}{s}}{rs} = \frac{\frac{s}{s}\cdot\frac{1}{r}-\frac{1}{s}\cdot\frac{r}{r}}{rs}$$

$$= \frac{\frac{s-r}{rs}}{rs}$$

$$= \frac{s-r}{rs} \div rs$$

$$= \frac{s-r}{rs} \div \frac{rs}{1}$$

$$= \frac{s-r}{rs} \cdot \frac{1}{rs} \quad = \frac{s-r}{r^2s^2}$$

6. $$\frac{(x-3)+\frac{2}{x}}{(x-4)+\frac{3}{x}} = \frac{\frac{(x-3)}{1}+\frac{2}{x}}{\frac{(x-4)}{1}+\frac{3}{x}}$$

$$= \frac{\frac{x}{x}\cdot\frac{(x-3)}{1}+\frac{2}{x}}{\frac{x}{x}\cdot\frac{(x-4)}{1}+\frac{3}{x}}$$

$$= \frac{\frac{x^2-3x+2}{x}}{\frac{x^2-4x+3}{x}}$$

$$= \frac{x^2-3x+2}{x} \div \frac{x^2-4x+3}{x}$$

$$= \frac{x^2-3x+2}{x} \cdot \frac{x}{x^2-4x+3}$$

$$= \frac{(x-2)\cancel{(x-1)}}{\cancel{x}} \cdot \frac{\cancel{x}}{(x-3)\cancel{(x-1)}}$$

$$= \frac{x-2}{x-3}$$

We will consider some examples that have negative exponents as well. The key is to rewrite them with positive exponents first. Remember, you cannot move stuff with negative exponents across the fraction bar when it is attached by addition or subtraction. You can only do this when you have multiplication between everything!

7. $$\frac{x^{-1}+y^{-1}}{(x+y)^{-1}}=\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x+y}}$$

Rewrite without negative exponents. Note that we did not move the x^{-1} or the y^{-1} across the main fraction bar. This is because they are attached with addition, not multiplication.

$$=\frac{\frac{y}{y}\cdot\frac{1}{x}+\frac{1}{y}\cdot\frac{x}{x}}{\frac{1}{x+y}}$$

$$=\frac{\frac{y+x}{xy}}{\frac{1}{x+y}}$$

$$=\frac{y+x}{xy}\div\frac{1}{x+y}$$

$$=\frac{y+x}{xy}\cdot\frac{x+y}{1}$$

Recall: $y+x=x+y$

$$=\frac{(x+y)^2}{xy}$$

8. $$\frac{x^{-3}-y^{-3}}{x^{-2}-y^{-2}}=\frac{\frac{1}{x^3}-\frac{1}{y^3}}{\frac{1}{x^2}-\frac{1}{y^2}}$$

Rewrite <u>without</u> negative exponents.

$$=\frac{\frac{y^3}{y^3}\cdot\frac{1}{x^3}-\frac{1}{y^3}\cdot\frac{x^3}{x^3}}{\frac{y^2}{y^2}\cdot\frac{1}{x^2}-\frac{1}{y^2}\cdot\frac{x^2}{x^2}}$$

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$$=\frac{\frac{y^3-x^3}{x^3y^3}}{\frac{y^2-x^2}{x^2y^2}}$$

$$=\frac{y^3-x^3}{x^3y^3}\div\frac{y^2-x^2}{x^2y^2}$$

$$=\frac{y^3-x^3}{x^3y^3}\cdot\frac{x^2y^2}{y^2-x^2}$$

$$=\frac{(\cancel{y-x})(y^2+xy+x^2)}{\cancel{x\cdot x}\cdot x\cdot\cancel{y\cdot y}\cdot y}\cdot\frac{\cancel{x\cdot x\cdot y\cdot y}}{(y+x)(\cancel{y-x})}$$

$$=\frac{y^2+xy+x^2}{xy(y+x)}$$