1.5 Factoring Trinomials

Recall that trinomials are polynomials that have three terms. There are a few methods that can be used to factor trinomials. One such method is the ac-method where the first and last terms are multiplied together and possibilities of factors that add up to the middle terms are considered. We prefer to use the "rainbow method" which uses trial and error to "unFOIL" the polynomial. There are other methods that do this as well, like the X method. This lesson is presented using the rainbow method and the ac-method, but if you prefer another method from a prior class, you may use that one. There are an excessive number of examples in this lesson to help students really catch on to their chosen technique. Factoring trinomials is a necessary skill for many topics in this course as well as in higher mathematics courses.

Let us begin by thinking about multiplication. We know how to multiply two binomials together:

$$\begin{aligned}(x+3)(x+5) &= x(x+5)+3(x+5)\\ &= x^2+\underbrace{5x+3x}+15\\ &= x^2+\quad 8x \quad +15\end{aligned}$$

Factoring is the "undoing" of this process (multiplication).

We will consider two methods for this example, the ac-method and the rainbow method.

<u>ac-method</u>

Let's work backward and attempt to factor the polynomial $x^2+8x+15$. Consider the middle term, $8x$. We want to split that up into two numbers that are integer factors of $ac = 1 \cdot 15 = 15$ that add to $8x$. (Recall, $ax^2 + bx + c$, so a=1 and b=15.) So, the integer factors of 15 are 1x15 and 3x5. Now, choose the pair of factors that have the sum $8x = 5x + 3x$. Replace the $8x$ with $5x + 3x$ and factor by grouping.

$$\begin{aligned} x^2 + 8x + 15 &= \underbrace{x^2 + 5x} + \underbrace{3x + 15} \\ &= x(x+5) + 3(x+5) \\ &= (x+5)(x+3) \end{aligned}$$

<u>Rainbow method</u>

Consider the FOIL method of multiplying two binomials together. The “F” represents the product of the first term in each binomial, x^2. So, to “undo” that multiplication, $x^2 = x \cdot x$. So, we will use x for the first term in each of the binomials. Next, we consider the “L” in FOIL that represents the product of the last term in each binomial, 15. So, the factors of 15 are 1x15 or 3x5. We look for the pair of factors that has a sum of 8 (the coefficient of the “middle” term of the trinomial). Since $3 + 5 = 8$, we choose those factors to complete the two binomials. Make sure to FOIL the two binomials so that you double check your work (especially the “sign”) to make sure you have the correct factorization.

$$\underbrace{x^2} + 8x + 15 = \left(\underbrace{x} \quad\quad\right)\left(\underbrace{x} \quad\quad\right)$$

Focus on the first term and break it down into the first two terms.

$$= (x \quad 3)(x \quad 5)$$

Next, focus on the last term and break it down into the last two terms. There will be some trial and error in this process. Just make a choice and try it. At this point, do not consider the signs.

$$= (x \quad 3)(x \quad 5)$$

$3x$

$5x$

Now, multiply the inners and the outers. This will allow you to assign positives and negatives in such a way that they add up to the middle. If you cannot get them to add up, then you try another factorization.

$$= (x \quad 3)(x \quad 5)$$

$+3x$

$+5x$

If we assign both to be positive, they will add up to the middle, $+8x$. The top sign goes to the first factor and the bottom sign goes to the second factor.

$$= (x + 3)(x + 5)$$

$+3x$

$+5x$

Double check that the signs multiply correctly. This is very important!

Examples

1. $x^2 + 5x + 6$

Using the rainbow method:

$$x^2 + 5x + 6 = (x \quad\quad)(x \quad\quad)$$

Focus on the first term and break it down into the first two terms.

$$= (x \quad 2)(x \quad 3)$$

Next, focus on the last term and break it down into the last two terms. There will be some trial and error in this process. Just make a choice and try it. At this point, do not consider the signs.

$$= (x \quad 2)(x \quad 3)$$

$2x$

$3x$

Now, multiply the inners and the outers. This will allow you to assign positives and negatives in such a way that they add up to the middle. If you cannot get them to add up, then you try another factorization.

$$= (x \quad \underbrace{2)(x}_{+2x} \quad 3)$$

$+3x$

If we assign both to be positive, they will add up to the middle, $+5x$. The top sign goes to the first factor and the bottom sign goes to the second factor.

$$= (x + \underbrace{2)(x}_{+2x} + 3)$$

$+3x$

Double check that the signs multiply correctly to get the last term.

Using the ac-method:

The third term is 6.

Factors of 6	**Sums of the Factors**	
1 and 6	$1 + 6 = 7$	
-1 and -6	$-1 + (-6) = -7$	
2 and 3	$2 + 3 = 5$	This is the correct pair!
-2 and -3	$-2 + (-3) = -5$	

So use these two numbers to make the middle and factor by grouping:

$$x^2 + 5x + 6 = \underbrace{x^2 + 2x} + \underbrace{3x + 6}$$

$$= x(x + 2) + 3(x + 2)$$

$$= (x + 2)(x + 3)$$

2. $x^2 - 5x + 6$

Using the rainbow method:

$$x^2 - 5x + 6 = (x \quad)(x \quad)$$

Focus on the first term and break it down into the first two terms.

$$= (x \quad 2)(x \quad 3)$$

Next, focus on the last term and break it down into the last two terms. There will be some trial and error in this process. Just make a choice and try it. At this point, do not consider the signs.

$$= (x \quad 2)(x \quad 3)$$

$2x$

$3x$

Now, multiply the inners and the outers. This will allow you to assign positives and negatives in such a way that they add up to the middle. If you cannot get them to add up, then you try another factorization.

$$= (x \quad 2)(x \quad 3)$$

$-2x$

$-3x$

If we assign both to be negative, they will add up to the middle, $-5x$. The top sign goes to the first factor and the bottom sign goes to the second factor.

$$= (x - 2)(x - 3)$$

$-2x$

$-3x$

Double check that the signs multiply correctly to get the last term.

Using the ac-method:

The third term is 6.

Factors of 6	**Sums of the Factors**	
1 and 6	$1+6=7$	
-1 and -6	$-1+(-6)=-7$	
2 and 3	$2+3=5$	
-2 and -3	$-2+(-3)=-5$	This is the correct pair!

So use these two numbers to make the middle and factor by grouping:

$$x^2-5x+6=\underbrace{x^2-2x}+\underbrace{3x-6}$$

$$=x(x-2)+3(x-2)$$

$$=(x-2)(x+3)$$

3. x^2+5x-6

Using the rainbow method:

Focus on the first term and break it down into the first two terms.

$$x^2+5x-6=(x\qquad)(x\qquad)$$

Next, focus on the last term and break it down into the last two terms. Notice that if you try 2 and 3, you cannot get the double check at the end to work out with the multiplication of signs. This is how you know that you need to try a different pair.

$$=(x\quad 6)(x\quad 1)$$

$$= (x \quad 6)(x \quad 1)$$

$$6x$$

$$x$$

Now, multiply the inners and the outers. This will allow you to assign positives and negatives in such a way that they add up to the middle. If you cannot get them to add up, then you try another factorization.

$$= (x \quad 6)(x \quad 1)$$

$$6x$$

$$-x$$

If we assign the larger number to be positive and the smaller one to be negative, they will add up to the middle, $+5x$. The top sign goes to the first factor and the bottom sign goes to the second factor.

$$= (x + 6)(x - 1)$$

$$+6x$$

$$-x$$

Double check that the signs multiply correctly to get the last term.

Using the ac-method:

The third term is -6.

Factors of -6	**Sums of the Factors**	
1 and -6	$1 + (-6) = -5$	
-1 and 6	$-1 + 6 = 5$	This is the correct pair!
2 and -3	$2 + (-3) = -1$	
-2 and 3	$-2 + 3 = 1$	

So use these two numbers to make the middle and factor by grouping:

$$x^2 + 5x - 6 = \underbrace{x^2 - x} + \underbrace{6x - 6}$$

$$= x(x - 1) + 6(x - 1)$$

$$= (x - 1)(x + 6)$$

4. $x^2 - 5x - 6$

Using the rainbow method:

$$x^2 - 5x - 6 = (x \quad)(x \quad)$$

Focus on the first term and break it down into the first two terms.

$$= (x \quad 6)(x \quad 1)$$

Next, focus on the last term and break it down into the last two terms. Notice that if you try 2 and 3, you cannot get the double check at the end to work out with the multiplication of signs. This is how you know that you need to try a different pair.

$$= (x \quad 6)(x \quad 1)$$

$6x$

x

Now, multiply the inners and the outers. This will allow you to assign positives and negatives in such a way that they add up to the middle. If you cannot get them to add up, then you try another factorization.

$$= (x \quad 6)(x \quad 1)$$

$-6x$

$+x$

If we assign the larger number to be negative and the smaller one to be positive, they will add up to the middle, $-5x$. The top sign goes to the first factor and the bottom sign goes to the second factor.

$$= (x - 6)(x + 1)$$

$-6x$

$+x$

Double check that the signs multiply correctly to get the last term.

Using the ac-method:

The third term is -6.

Factors of -6	Sums of the Factors	
1 and -6	$1 + (-6) = -5$	This is the correct pair!
-1 and 6	$-1 + 6 = 5$	
2 and -3	$2 + (-3) = -1$	
-2 and 3	$-2 + 3 = 1$	

So use these two numbers to make the middle and factor by grouping:

$$x^2 - 5x - 6 = \underbrace{x^2 + x} - \underbrace{6x - 6}$$

$$= x(x+1) - 6(x+1)$$

$$= (x+1)(x-6)$$

5. $9x^2 - 18x + 5$

Using the rainbow method:

$$9x^2 - 18x + 5 = (3x \quad\quad)(3x \quad\quad)$$

Focus on the first term and break it down into the first two terms. I usually begin with "middle of the road" values if the middle term is not incredibly large compared to the end terms.

$$= (3x \quad 5)(3x \quad 1)$$

Next, focus on the last term and break it down into the last two terms. Here, we don't have much of a choice, since 5 is prime.

$$= (3x \quad 5)(3x \quad 1)$$

$15x$

$3x$

Now, multiply the inners and the outers. This will allow you to assign positives and negatives in such a way that they add up to the middle. If you cannot get them to add up, then you try another factorization. Here, we can get them to add up to 18x.

$$= (3x \quad 5)(3x \quad 1)$$

$-15x$

$-3x$

If we assign them both to be negative, they will add up to $-18x$. The top sign goes to the first factor and the bottom sign goes to the second factor.

$$= (3x - 5)(3x - 1)$$

$-15x$

$-3x$

Double check that the signs multiply correctly to get the last term.

Using the ac-method:

Since the first term has a coefficient other than 1 (as seen previously), we will need to consider the factors of the coefficients of the first and the third terms that are 9 and 5, respectively. When we list the factors of the leading coefficient, we will only list the positive factors. We will then proceed using trial and check to find the correct factorization.

Factors of 9		**Factors of 5**	
1 and 9	3 and 3	1 and 5	-1 and -5

It is easiest to begin with the pair of factors for the first term that are the closest together on the number line. If the number in the middle is "not too large" compared with the numbers on the end, this will often be the correct combination. It is a very efficient way to approach the problem. So, I will use the pair “3 and 3” for the first term in the binomials and then I will use -1 and -5 for the pair of factors to fill the last term in the binomials since this is the only way to get a negative middle term in the trinomial. (Note: If the number in the middle were really large (or small) compared to the number on the ends, it might be more efficient to start with the “extreme” numbers instead.)

Let’s try $(3x-1)(3x-5)$. We must check by multiplying the outer and the inner parts of the F**OI**L process to verify the middle term will be the same as your -18x as given in the question. The outer product is $3x(-5) = -15x$ and the inner product is $(-1)(3x) = -3x$. If we combine the two products, we get $-15x \pm 3x = -18x$ which is exactly what we wanted.

$$9x^2 - 18x + 5 = (3x-1)(3x-5)$$

6. $7x^2 - 58x - 45$

Using the rainbow method:

Focus on the first term and break it down into the first two terms.

$$7x^2 - 58x - 45 = (7x \qquad)(x \qquad)$$

Next, focus on the last term and break it down into the last two terms. Here, the “middle of the road” choice is 9 times 5 and you may need to switch their order to get the middle if your first attempt does not do it.

$$= (7x \quad 5)(x \quad 9)$$

$$= (7x \quad 5)(x \quad 9)$$

$5x$

$63x$

Now, multiply the inners and the outers. This will allow you to assign positives and negatives in such a way that they add up to the middle. If you cannot get them to add up, then you try another factorization. Here, we can get them to add up to -58x.

$$= (7x \quad 5)(x \quad 9)$$

$5x$

$-63x$

If we assign the larger one to be negative, they will add up to $-58x$. The top sign goes to the first factor and the bottom sign goes to the second factor.

$$= (7x + 5)(x - 9)$$

$+5x$

$-63x$

Double check that the signs multiply correctly to get the last term.

Using the ac-method:

The first term has a coefficient of 7 and the third has a coefficient of -45.

<u>Factors of 7</u>	**<u>Factors of -45</u>**	
1 and 7	-1 and 45	1 and -45
	-3 and 15	3 and -15
	-5 and 9	5 and -9

We will start with 1 and 7 (the only choice for the coefficients of the first terms) and we will start at the bottom of the list with -5 and 9 for the last terms. Then, we will check the outer and inner products of FOIL. We want $-58x$.

$(x-5)(7x+9)$

outer $= x \cdot 9 = 9x$ inner $= -5 \cdot 7x = -35x$

sum of outer/inner products $= 9x + (-35x) = -26x$

Since this is not $-58x$, we can "switch" the -5 and the 9.

$(x+9)(7x-5)$

outer $= x \cdot -5 = -5x$ inner $= 9 \cdot 7x = 63x$

sum of outer/inner products $= -5x + 63x = 58x$

We need it to be negative, so we must "switch" the signs and check.

$(x-9)(7x+5)$

outer $= x \cdot 5 = 5x$ inner $= -9 \cdot 7x = -63x$

sum of outer/inner products $= 5x + (-63x) = -58x$

$$7x^2 - 58x - 45 = (x-9)(7x+5)$$

7. $19x + 6x^2 - 36$ Put this in descending order before factoring.

$6x^2 + 19x - 36$

Using the rainbow method:

$$6x^2 + 19x - 36 = (2x \qquad)(3x \qquad)$$

Focus on the first term and break it down into the first two terms. Here the "middle of the road" choice is 2 times 3, so we will try it first.

$$= (2x \quad 9)(3x \quad 4)$$

Next, focus on the last term and break it down into the last two terms. Here, the "middle of the road" choice is 9 times 4 and you may need to switch their order to get the middle if your first attempt does not do it.

$$= (2x \quad 9)(3x \quad 4)$$

$27x$

$8x$

Now, multiply the inners and the outers. This will allow you to assign positives and negatives in such a way that they add up to the middle. If you cannot get them to add up, then you try another factorization. Here, we can get them to add up to +19x.

$$= (2x \quad 9)(3x \quad 4)$$

$+27x$

$-8x$

If we assign the larger one to be positive, they will add up to $+19x$. The top sign goes to the first factor and the bottom sign goes to the second factor.

$$= (2x + 9)(3x - 4)$$

$+27x$

$-8x$

Double check that the signs multiply correctly to get the last term.

Using the ac-method:

$6x^2 + 19x - 36$

Factors of 6	**Factors of -36**	
1 and 6	-1 and 36	1 and -36
2 and 3	-2 and 18	3 and -18
	-3 and 12	3 and -12
	-4 and 9	4 and -9
	-6 and 6	*(6 and -6 repeat -6 and 6)

We will start with 2 and 3 for the first term and -6 and 6 for the last term. We want $19x$ for the middle term.

$(2x-6)(3x+6)$

outer $= 2x \cdot 6 = 12x$ inner $= -6 \cdot 3x = -18x$

sum of outer/inner products $= 12x + (-18x) = -6x$

This is not $19x$, so I will try another pair.

Note: It is possible to determine that this trial would fail in the beginning, since both binomials had a common factor that could have been factored out. If this were the case, we should have been able to factor out a GCF in the beginning!

$(2x-4)(3x+9)$ This trial is using -4 and 9 for the last term.

outer $= 2x \cdot 9 = 18x$ inner $= -4 \cdot 3x = -12x$

sum of outer/inner products $= 18x + (-12x) = 6x$

This is not $19x$, so I will "switch" the -4 and 9.

$(2x+9)(3x-4)$

outer $= 2x \cdot -4 = -8x$ inner $= 9 \cdot 3x = 27x$

sum of outer/inner products $= -8x + 27x = 19x$

This is $19x$, so we have the correct factorization.

$$6x^2 + 19x - 36 = (x-9)(7x+5)$$

8. $-25x + 28x^2 - 8$ Put this in descending order before factoring.

$28x^2 - 25x - 8$

Using the rainbow method:

Focus on the first term and break it down into the first two terms that you are planning to try first.

$28x^2 - 25x - 8 = (7x \quad)(4x \quad)$

Next, focus on the last term and break it down into the last two terms. Here, the "middle of the road" choice is 2 times 4, but we can see that those will not work here since if we use them we will be able to factor at least a 2 out of the second factor (then we should have been able to factor that out in the beginning as a GCF). So we try 8 times 1 instead.

$= (7x \quad 8)(4x \quad 1)$

$$= (7x \quad 8)(4x \quad 1)$$

$32x$

$7x$

Now, multiply the inners and the outers. This will allow you to assign positives and negatives in such a way that they add up to the middle. If you cannot get them to add up, then you try another factorization. Here, we can get them to add up to -25x.

$$= (7x \quad 8)(4x \quad 1)$$

$-32x$

$+7x$

If we assign the larger one to be negative, they will add up to $-25x$. The top sign goes to the first factor and the bottom sign goes to the second factor.

$$= (7x - 8)(4x + 1)$$

$-32x$

$+7x$

Double check that the signs multiply correctly to get the last term.

Using the ac-method:

$28x^2 - 25x - 8$

Factors of 28	**Factors of -8**	
1 and 28	-1 and 8	1 and -8
2 and 14	-2 and 4	2 and -4
4 and 7		

We will start with 4 and 7 for the first term and -2 and 4 for the last term. We want $-25x$ for the middle term.

$(4x - 2)(7x + 4)$

outer $= 4x \cdot 4 = 16x$ inner $= -2 \cdot 7x = -14x$

sum of outer/inner products $= 16x + (-14x) = 2x$

Since this didn't work, we will "switch" -2 and 4.

$(4x + 4)(7x - 2)$
outer $= 4x \cdot -2 = -8x$ inner $= 4 \cdot 7x = 28x$
sum of outer/inner products $= -8x + 28x = 20x$
Since this didn't work, we will try -1 and 8 for the last term.

$(4x - 1)(7x + 8)$
outer $= 4x \cdot 8 = 32x$ inner $= -1 \cdot 7x = -7x$
sum of outer/inner products $= 32x + (-7x) = 25x$
We are close! The "sign" should be negative, so "switch" signs.

$(4x + 1)(7x - 8)$
outer $= 4x \cdot -8 = -32x$ inner $= 1 \cdot 7x = 7x$
sum of outer/inner products $= -32x + 7x = -25x$
This is $-25x$, so we have the correct factorization.

$$28x^2 - 25x - 8 = (4x + 1)(7x - 8)$$

Sometimes we will encounter a polynomial which has a common factor to all of the terms. We should always be sure to check for this and to pull out any common factor from ALL terms before factoring further. This will make our numbers much easier to work with when factoring as well. We will now consider examples where we must pull out a GCF.

9. $-20x^3 - 58x^2 + 42x$ Always factor out a GCF first, if possible.
$= -2x(10x^2 + 29x - 21)$ Now, factor the trinomial.

Note: $-2x$ is a factor of the original trinomial and is part of the answer.

Using the rainbow method:

$$10x^2 + 29x - 21 = \quad (5x \qquad)(2x \qquad)$$

$$= (5x \quad 3)(2x \quad 7)$$

$$= (5x \quad 3)(2x \quad 7)$$

$6x$

$35x$

Note that it may take several attempts to get the numbers you need to add up to the middle term. If you tried the 7 and the 3 in reverse order, you might think it worked until you double-checked the sign of the product at the last step! Gladly, each attempt is very quick and there are only finitely many combinations that you can try.

$$= (5x \quad 3)(2x \quad 7)$$

$-6x$

$+35x$

$$= (5x - 3)(2x + 7)$$

$-6x$

$+35x$

Don't forget that $-2x$ is also a factor, so include it in your answer:
$-2x(5x - 3)(2x + 7)$

Using the ac-method:

Factors of 10	**Factors of -21**	
1 and 10	-1 and 21	1 and -21
2 and 5	-3 and 7	3 and -7

We will start with 2 and 5 for the first term and -3 and 7 for the last term. We want $29x$ for the middle term.

$(2x - 3)(5x + 7)$
outer $= 2x \cdot 7 = 14x$ inner $= -3 \cdot 5x = -15x$
sum of outer/inner products $= 14x + (-15x) = x$

Since this didn't work, we will "switch" -3 and 7.

$(2x+7)(5x-3)$
outer $= 2x \cdot -3 = -6x$ inner $= 7 \cdot 5x = 35x$
sum of outer/inner products $= -6x + 35x = 29x$
This is $29x$, so we have the correct factorization.

$$-20x^3 - 58x^2 + 42x = -2x(2x+7)(5x-3)$$

10. $18x^4z + 15x^3z - 75x^2z$ Factor out GCF first.
$= 3x^2z(6x^2 + 5x - 25)$ Now, factor the trinomial.

Using the rainbow method:

$6x^2 + 5x - 25 \quad = (2x \quad\quad)(3x \quad\quad)$

$= (2x \quad 5)(3x \quad 5)$

$= (2x \quad 5)(3x \quad 5)$

$15x$

$10x$

$= (2x \quad 5)(3x \quad 5)$

$+15x$

$-10x$

$= (2x + 5)(3x - 5)$

$+15x$

$-10x$

Don't forget that $3x^2z$ is also a factor, so include it in your answer:
$3x^2z(2x+5)(3x-5)$

Using the ac-method:

Factors of 6	**Factors of -25**	
1 and 6	-1 and 25	1 and -25
2 and 3	-5 and 5	

We will start with 2 and 3 for the first term and -5 and 5 for the last term. We want $5x$ for the middle term.

$(2x-5)(3x+5)$
outer $=2x\cdot 5=10x$ inner $=-5\cdot 3x=-15x$
sum of outer/inner products $=10x+(-15x)=-5x$
But, we need positive $5x$, so we "switch" the sign.

$$18x^4z+15x^3z-75x^2z=3x^2z(2x+5)(3x-5)$$

11. $9b^4-48b^2c^2+64c^4$

Using the rainbow method:

$$9b^4-48b^2c^2+64c^4=(3b^2\qquad)(3b^2\qquad)$$

$$=(3b^2\quad 8c^2)(3b^2\quad 8c^2)$$

$$=(3b^2\quad 8c^2)(3b^2\quad 8c^2)$$

$24b^2c^2$

$24b^2c^2$

$$=(3b^2\quad 8c^2)(3b^2\quad 8c^2)$$

$-24b^2c^2$

$-24b^2c^2$

$$= (3b^2 - 8c^2)(3b^2 - 8c^2)$$

$-24b^2c^2$

$-24b^2c^2$

OR $\quad = (3b^2 - 8c^2)^2$

Using the ac-method:

<u>**Factors of 9**</u>	<u>**Factors of 64**</u>	
1 and 9	1 and 64	-1 and -64
3 and 3	2 and 32	-2 and -32
	4 and 16	-4 and -16
	8 and 8	-8 and -8

We will start with 3 and 3 for the first term and -8 and -8 for the last term. We do not need to try any of the positive factors for the last term since the middle term is negative. We want $-48b^2c^2$ for the middle term.

$(3b^2 - 8c^2)(3b^2 - 8c^2) = (3b^2 - 8c^2)^2$

outer $= 3b^2 \cdot -8c^2 = -24b^2c^2$
inner $= 3b^2 \cdot -8c^2 = -24b^2c^2$

sum of outer/inner products $= -24b^2c^2 + (-24b^2c^2) = -48b^2c^2$

$9b^4 - 48b^2c^2 + 64c^4 = (3b^2 - 8c^2)(3b^2 - 8c^2) = (3b^2 - 8c^2)^2$

When a trinomial factors into the square of a binomial it is a special type of trinomial called a Perfect Square Trinomial.

(Note that when the middle term is double the product of the square roots of the first and last terms, you can tell that you have a perfect square trinomial. For this reason, you could use a formula if you recognize the pattern, but you can also use the regular method of your choice and you will still arrive at the same answer!)

Perfect Square Trinomial Formulas:

$\boldsymbol{(a+b)^2 = a^2 + 2ab + b^2}$

$\boldsymbol{(a-b)^2 = a^2 - 2ab + b^2}$

To identify a perfect square trinomial, we look for the coefficient of the first and last terms to be a perfect square. We take the square root of both terms and keep the sign of the middle term. Make sure to check!
We will do the next example using the perfect square trinomial formulas.

12. $x^2 + 18x + 81$

First, we can see that the first and last terms are perfect squares. If we take the square root of each and double it, we see that we get the middle term, so this is indeed a perfect square trinomial and we can use the formula above.

$a = \sqrt{x^2} = x$ and $b = \sqrt{81} = 9$

So, $x^2 + 18x + 81 = (x+9)^2$.

(To check the middle term, multiply $2ab = 2 \cdot x \cdot 9 = 18x$.)

Sometimes, we could use a substitution to make our polynomial easier to factor. We will consider the next example this way, although you could multiply it all out prior to factoring and you will get the same answer. Substitution is much faster!

13. $(a-7)^2 - 9(a-7) - 36$

We can substitute for $(a-7)$ to make it easier to factor. Let $y = (a-7)$.

So, $(a-7)^2 - 9(a-7) - 36 \Longrightarrow y^2 - 9y - 36$
$= (y-12)(y+3)$

Now, "back substitute" and replace y with $a - 7$, then simplify.

$$= (a-7-12)(a-7+3)$$
$$= (a-19)(a-4)$$

There is another way (expand using multiplication):

$$\begin{aligned} &\underbrace{(a-7)^2} \quad \underbrace{-9(a-7)} \quad -36 \\ &= a^2 - 14a + 49 \quad -9a + 63 \quad -36 \\ &= a^2 - 23a + 76 \\ &= (a-4)(a-19) \end{aligned}$$

*Note: The factors of 76 are 1 and 76, 2 and 38, 4 and 19.
Since -4 + -19 = -23, they are the factors required.