1.2 Integer Exponents

Since we have already learned how to solve *linear* equations (equations with an exponent of 0 or 1), we would like to be able to solve equations that have higher powers in them, but before we can do that, we need to know how to "work with" expressions that contain *exponents*. In order to solve equations, sometimes we need to simplify the expressions in the equations prior to isolating the variable. This is why we will always explore the operations on the expressions before we delve into solving the equations that contain those expressions. This lesson on exponents will prepare us for working with *polynomials* in the next few lessons and then finally we will solve *polynomial* equations (at least some types of them). But this lesson will also prepare us for future lessons with radicals and rational exponents, as the same rules that apply here will apply there. This lesson is foundational, just as the last lesson was, and it is worth spending the time to really understand the rules of exponents and be able to work with them. We will begin with the definition of an exponent.

n is called the "exponent" (or power)

Definition: $b^n = \underbrace{b \cdot b \cdot b \cdot \ldots \cdot b}$

b is called the "base" "*n*" of these

Examples

Evaluate each of the following exponential expressions.

1. $5^2 = 5 \cdot 5 = 25$

2. $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

Exponent Rules

1. $b^m \cdot b^n = b^{m+n}$

Example: $x^3 \cdot x^2 = x^{3+2} = x^5$

Why?

$2^3 \cdot 2^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

$8 \cdot 4$

32

2. $\frac{b^m}{b^n} = b^{m-n}$

Example: $\frac{y^{15}}{y^8} = y^{15-8} = y^7$

Why?

$\frac{2^5}{2^3} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$

$\frac{32}{8} = 2^2$

$4 \quad 4$

3. $(b^m)^n = b^{mn}$

Example: $(x^4)^3 = x^{12}$

Why?

$(2^3)^2 = 2^3 \cdot 2^3$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 64$

$8^2 = 64$

4. $(a \cdot b)^m = a^m \cdot b^m$

Example: $(3x^5)^2 = 3^2(x^5)^2$
$= 9x^{10}$

Why?

$(2 \cdot 5)^2 = (2 \cdot 5)(2 \cdot 5)$

$10^2 = 2 \cdot 5 \cdot 2 \cdot 5$

$100 = 2 \cdot 2 \cdot 5 \cdot 5$

$= 2^2 \cdot 5^2$

$= 4 \cdot 25$

$= 100$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Example: $\left(\frac{-2x}{y^3}\right)^2 = \frac{(-2x)^2}{(y^3)^2}$

$= \frac{4x^2}{y^6}$

Why?

$\left(\frac{8}{4}\right)^2 = \frac{8^2}{4^2}$

$2^2 = \frac{64}{16}$

$4 = 4$

6. $b^0 = 1$

Example: $(-3x^{18}y^2z^{-52})^0 = 1$

<u>Why?</u>

$$
\begin{aligned}
(2)^5 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \underbrace{32 \quad \div 2} \\
(2)^4 &= 2 \cdot 2 \cdot 2 \cdot 2 \qquad = \underbrace{16 \quad \div 2} \\
(2)^3 &= 2 \cdot 2 \cdot 2 \qquad\quad = \underbrace{8 \quad \div 2} \\
(2)^2 &= 2 \cdot 2 \qquad\qquad = \underbrace{4 \quad \div 2} \\
(2)^1 &= 2 \qquad\qquad\quad\; = \underbrace{2 \quad \div 2} \\
(2)^0 & \qquad\qquad\qquad\quad\;\; = 1
\end{aligned}
$$

7. $b^{-n} = \frac{1}{b^n}$

Why?

It is a definition.

<u>Examples</u>

I. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

II. $4x^{-3} = \frac{4}{x^3}$

III. $(4x)^{-3} = \frac{1}{(4x)^3} = \frac{1}{4^3x^3} = \frac{1}{64x^3}$

IV. $\frac{5}{2^{-3}} = 5 \cdot 2^3 = 5 \cdot 8 = 40$

<u>Examples</u> (Putting it all together.)

Simplify each exponential expression completely. Answers should contain only positive exponents.

1. $\frac{(2x^{-2})^3}{(3x)^{-2}}$	1. Outermost exponent (move whole piece if negative)
$= (2x^{-2})^3(3x)^2$	2. Distribute outermost exponent (if possible, multiply/divide)
$= 8x^{-6} \cdot 9x^2$	3. Clean up (adding exponents, multiplying numbers, etc.)
$= 8 \cdot 9 \cdot x^{-6} \cdot x^2$	
$= 72x^{-4}$	4. Move any stuff with negative exponents
$= \frac{72}{x^4}$	

2. $\frac{-12x^{-9}y^{10}}{4x^{-12}y^7} = -3x^3y^3$

We can do this one by either subtracting exponents of the same variable (using Rule 3)

$$\frac{-12x^{-9}y^{10}}{4x^{-12}y^7} = -3x^{-9-(-12)}y^{10-7} = -3x^3y^3$$

Or we can simplify by moving exponents and then adding them on the same side of the fraction bar (using Rules 7 and 2).

$$\frac{-12x^{-9}y^{10}}{4x^{-12}y^{7}} = -3x^{-9}x^{12}y^{10}y^{7} = -3x^{3}y^{3}$$

We will always get the same answer if we apply the rules correctly!

3. $\left(\frac{-3x^2y^{-1}z}{2x^3yz^2}\right)^{-2}$

Note: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$

This is because:

$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{a}{b}\right)^{n}$

$= \left(\frac{2x^3yz^2}{-3x^2y^{-1}z}\right)^{2}$

Make sure that the exponent is distributed to all the numbers and all the variables.

$= \frac{4x^6y^2z^4}{9x^4y^{-2}z^2}$

$= \frac{4x^2y^4z^2}{9}$

4. $(2x^{-1}y^3)^2(4x^5y^{-2})^{-3}$

Remove outermost negative exponent by moving whole piece to denominator

$= \frac{(2x^{-1}y^3)^2}{(4x^5y^{-2})^3}$

Distribute exponents

$= \frac{4x^{-2}y^6}{64x^{15}y^{-6}}$

Move factors with negative exponents

$= \frac{4y^6y^6}{64x^{15}x^2}$

Clean it up (combine variables and reduce numbers, etc.)

$= \frac{y^{12}}{16x^{17}}$