## 1.1 Basic Equation Solving

The first part of this course focuses on equation solving. You will learn about many types of equations and techniques for solving them, some of which you are already familiar with and some of which will probably be new to you. The types of equations that are covered in this course are polynomial (including linear and quadratic), absolute value (linear only), rational, radical, exponential, and logarithmic. There is a common thread to equation solving that involves isolating the variable, or getting the variable alone on one side of the equation by "undoing" whatever has been done to it, making sure to perform the action on both sides of the equation! It is much like unwrapping a present. You work from the outside in, peeling off the layers around the variable until it is isolated. Since you are "undoing", you typically work in the *reverse* order of operations (except that sometimes you eliminate the parentheses before you begin). When there is a single variable in the equation, this a fairly straightforward thing to do, but when there is more than one expression involving the variable, sometimes we need to use another technique to make the equation simpler to solve (such as factoring or using a substitution).

In this lesson, we will explore the most basic types of equations upon which we will build our methods for more complicated equations – linear equations. You will know an equation is *linear* if the power, or exponent, on the variable is either 0 or 1, but no higher. These types of equations are called *linear* because they represent *lines* when you bring in another variable and graph them in two dimensions (we will do this later in the graphing portion of the course). We will also look at simple equations with more than one variable and practice isolating a given variable by treating the other variables like numbers. Toward the end of the lesson, we consider absolute value equations (linear only, although the technique can be applied to other kinds), where we need to split out the equation into two linear equations in order to obtain all of the solutions.

I. Linear Equations ("undoing" what has been done to the variable.)

Linear equations are the simplest kind to solve. At most we need to bring the variables to one side and then undo any addition/ subtraction and multiplication/division that have been done to the variable. To "undo" addition, we must subtract. To "undo" subtraction, we must add. To "undo" multiplication, we must divide. To "undo" division, we must multiply. Don't forget the Golden Rule of Equation Solving: Whatever you do to one side of the equation (as far as operations go), you must also do to the other side! Remember, you don't want to change it – you just want to rewrite it at each step.

**Examples**

**Solve each of the following equations for the indicated variable.**

1. $x + 7 = 33$ To "undo" adding 7 to the variable, subtract 7

$$\begin{array}{rcl} x + 7 & = & 33 \\ -7 & & -7 \\ \hline x & = & 26 \end{array}$$

2. $y - 4 = 5$ To "undo" subtracting 4 from the variable, add 4

$$\begin{array}{rcl} y - 4 & = & 5 \\ +4 & & +4 \\ \hline y & = & 9 \end{array}$$

3. $-3x = 12$ To "undo" multiplying the variable by -3, divide by -3

$$\frac{-3x}{-3} = \frac{12}{-3}$$

$$x = -4$$

4. $8y + 5 = 3y - 10$ To "isolate the variable", subtract 3y

$$\begin{array}{rcl} 8y + 5 & = & 3y - 10 \\ -3y \phantom{+5} & & -3y \\ \hline 5y + 5 & = & -10 \\ -5 & = & -5 \\ \hline 5y & = & -15 \\ \frac{5y}{5} & = & \frac{-15}{5} \\ y & = & -3 \end{array}$$

Next, subtract 5 to "isolate the variable"

To "undo" multiplying by 5, divide by 5

5. $-3(4y - 5) - y = -2(y - 9)$ Distribute

$$\begin{array}{rcl} -12y + 15 \ - y & = & -2y + 18 \\ \hline -13y + 15 & = & -2y + 18 \\ +2y \phantom{+15} & & +2y \\ \hline -11y + 15 & = & 18 \\ -15 & & -15 \\ \hline -11y & = & 3 \\ \frac{-11y}{-11} & = & \frac{3}{-11} \\ y & = & -\frac{3}{11} \end{array}$$

Next, combine like terms

Now, "isolate the variable"

To "undo" adding 15, subtract 15

Divide by -11

6. For this problem, we can approach it in two different ways.

<u>Method #1</u>

$$\frac{x}{3} + \frac{4}{5} = \frac{8}{15}$$

Get a common denominator (which is 15)

$$\frac{5}{5} \cdot \frac{x}{3} + \frac{4}{5} \cdot \frac{3}{3} = \frac{8}{15}$$

$$\frac{5x}{15} + \frac{12}{15} = \frac{8}{15}$$

Now, multiply by 15 to "clear the fraction"

$$15\left(\frac{5x}{15} + \frac{12}{15}\right) = \frac{8}{15} \cdot 15$$

$$5x + 12 = 8$$

Subtract 12

$$\begin{array}{rcl} & -12 & \quad -12 \\ \hline 5x & & = -4 \end{array}$$

Divide by 5

$$\frac{5x}{5} = \frac{-4}{5}$$

$$x = -\frac{4}{5}$$

<u>Method #2</u>

$$\frac{x}{3} + \frac{4}{5} = \frac{8}{15}$$

$$\begin{array}{rcl} -\frac{4}{5} & & -\frac{4}{5} \\ \hline \frac{x}{3} & = & -\frac{4}{15} \end{array}$$

Subtract $\frac{4}{5}$ from both sides:

$$\boxed{\frac{8}{15} - \frac{4}{5} = \frac{8}{15} - \frac{12}{15} = -\frac{4}{15}}$$

$$3\left(\frac{x}{3}\right) = \left(-\frac{4}{15}\right)3$$

$$x = -\frac{4}{5}$$

7. $0.2x + 0.1 = 0.12x - 0.06$ Multiply by 100 to "clear decimals".

$100(0.2x + 0.1) = (0.12x - 0.06)100$ Distribute 100

$$\begin{array}{rcl} 20x + 10 & = & 12x - 6 \\ -12x & = & -12x \\ \hline \end{array}$$
"Isolate the variable."

$$\begin{array}{rcl} 8x + 10 & = & -6 \\ -10 & = & -10 \\ \hline \end{array}$$
Subtract 10

$8x = -16$ Divide by 8

$$\frac{8x}{8} = \frac{-16}{8}$$

$$x = -2$$

II. Literal Equations (Solving for a specific variable.)

We will use the same techniques that we used previously in order to isolate the variable for a literal equation. The main difference between this type of equation and the ones encountered previously is that these equations have many variables in them and we will solve for one of those variables, treating the other variables as we do numbers. Sometimes the variable we are solving for occurs more than once in the equation and in that case we must get them all on one side and factor out the variable in order to isolate it. We are assuming some basic knowledge of factoring out a greatest common factor (GCF) here, which should be review, but we will cover it again in a future lesson.

## Examples

**Solve each of the following equations for the indicated variable.**

1. Solve for $x$:

$y = xt$ Since $xt$ indicates multiplication, we divide to isolate $x$.

$\frac{y}{t} = \frac{xt}{t}$

$\frac{y}{t} = x$ We can "switch" sides (if desired).

$x = \frac{y}{t}$

2. Solve for $R$:

$I = PRT$ Since $PRT$ indicates multiplication, divide to isolate $R$.

$\frac{I}{PT} = \frac{PRT}{PT}$

$\frac{I}{PT} = R$ "Switch" sides.

$R = \frac{I}{PT}$

3. Solve for $y$:

$k = ys - 7yz$ Since we are solving for "y", we want to isolate "y". But, there are two terms that contain y. So, we can factor out "y" from the two terms.

$k = y(s - 7z)$ Now, divide to "isolate y".

$$\frac{k}{(s-7z)} = \frac{y(s-7z)}{(s-7z)}$$

$$\frac{k}{(s-7z)} = y$$ "Switch" sides.

$$y = \frac{k}{(s-7z)}$$

4. Solve for *B*:

$$A = \frac{1}{2}(B+b)h$$ We must treat *B* and *b* as two different variables. First, multiply by 2 to clear fraction.

$$2 \cdot A = 2 \cdot \frac{1}{2}(B+b)h$$

$$2A = (B+b)h$$ Now, distribute *h*.

$$\begin{array}{rcl} 2A & = & Bh + bh \\ -bh & & -\ bh \\ \hline \end{array}$$ "Isolate" the term containing *B*, subtract *bh*.

$$2A - bh = Bh$$ Now, divide by *h* to isolate *B*.

$$\frac{2A-bh}{h} = \frac{Bh}{h}$$

$$\frac{2A-bh}{h} = B$$ "Switch" sides.

$$B = \frac{2A-bh}{h}$$

## III. Absolute Value Equations

There is more than one way to think of the concept of absolute value, but they really mean the same thing. One way is to simply "take the positive" of what is inside the absolute value symbols $| \quad |$.

For example, $|2| = 2$ and $|-2| = 2$.

We can think of absolute value as "distance from 0" as well. This means that 2 and -2 are both 2 units away from zero when we measure on the number line. To make this more formal mathematically, we could write the definition:

$$|a| = \begin{cases} a & if \;\; a > 0 \\ -a & if \;\; a < 0 \end{cases} \text{ (this measures the distance from 0).}$$

First, decide where $a$ is, and then you evaluate it. If $a$ is positive, then plug it into the first expression. If $a$ is negative, then plug it into the second expression. For example, $|-5| = -(-5) = 5$ since $-5 < 0$.

We are assuming a basic knowledge of the meaning of inequality symbols ($<$ means less than and $>$ means greater than) in order to understand this definition, but we will cover this later as well in the "Inequalities" portion of the course.

To solve absolute value equations, you **must** first isolate the absolute value. You can then make two equations.

Note: The first equation is made by dropping the absolute value bars. The second equation is made by dropping the absolute value bars and "changing" the sign on the other side (this means you are multiplying the other side by -1).

## Examples

**Solve each of the following equations for the indicated variable.**

1. $|x| = 2$ This question is asking "what numbers are 2 units away from 0?".

$|x| = 2$ Since the absolute value is isolated, we can make two equations.

$x = 2$ or $x = -2$ For the first equation, $x = 2$, we only **drop** the absolute value bars. For the second equation, $x = -2$, we **drop** the absolute value bars **AND change** the sign on the other side.

2. $|x + 3| = 7$ Since the absolute value is isolated, we can make two equations (just like we did above).

$$\begin{array}{rcl} x + 3 = 7 & \text{or} & x + 3 = -7 \\ -3 \quad -3 & & -3 \quad -3 \\ \hline x = 4 & \text{or} & x = -10 \end{array}$$

Now, we can solve each equation for *x*.

3. $|-3x + 1| = 10$

$$\begin{array}{rcl} -3x + 1 = 10 & \text{or} & -3x + 1 = -10 \\ -1 \quad -1 & & -1 \quad -1 \\ \hline -3x \quad = 9 & & -3x \quad = -11 \end{array}$$

$$\frac{-3x}{-3} = \frac{9}{-3} \qquad \frac{-3x}{-3} = \frac{-11}{-3}$$

$x = -3$ or $x = \frac{11}{3}$

4. $|-3x + 1| = -10$ No Solution

   If we take the absolute value of a number, either positive or negative, the result is positive. Since the absolute value is equal to $-10$, this is not possible.

5. $|x - 2| - 6 = -5$ We must "isolate" the absolute value first.

$$\begin{array}{rcl} |x-2| - 6 &=& -5 \\ +6 && +6 \\ \hline |x-2| &=& 1 \end{array}$$

Now, make two equations and solve.

$$\begin{array}{rcl} x - 2 &=& 1 \\ +2 && +2 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{rcl} x - 2 &=& -1 \\ +2 && +2 \\ \hline \end{array}$$

$x = 3$ or $x = 1$

Note: You may not distribute through absolute value bars!
You will need to multiply or divide to isolate the absolute value if it has a number in front of it. See the next example.

6. $2|3x-4|-8=-6$ “Isolate” the absolute value first.

$$\begin{array}{rcl} 2|3x-4| - 8 & = & -6 \\ +8 & & +8 \\ \hline 2|3x-4| & = & 2 \\ \frac{2|3x-4|}{2} & = & \frac{2}{2} \end{array}$$

$$|3x-4| \quad = 1$$

Now, make two equations and solve. **DO NOT** change anything inside the absolute value.

$$\begin{array}{rcl} 3x-4 = 1 & \text{or} & 3x-4=-1 \\ +4 \quad +4 & & +4 \quad +4 \\ \hline 3x = 5 & \text{or} & 3x = 3 \\ \frac{3x}{3} = \frac{5}{3} & \text{or} & \frac{3x}{3} = \frac{3}{3} \\ x = \frac{5}{3} & \text{or} & x = 1 \end{array}$$

Note: When there are two absolute values, one on each side of the equal sign, we will still make two equations in the same way that we have been. For the first equation, we will drop the bars. For the second equation, we will drop the bars and then change the sign on the other side by multiplying the other side by -1. This is because the “insides” could either be the same or they could be opposite signs of each other. See the next example.

7. $|2x+5| = |x-2|$ Make two equations.

$2x + 5 = x - 2$ or $2x + 5 = -1(x-2)$ Solve both equations.

$$\begin{array}{rcl} 2x + 5 &=& x - 2 \\ -x & & -x \\ \hline x + 5 &=& -2 \\ -5 & & -5 \\ \hline x &=& -7 \end{array}$$

$$\begin{array}{rcl} 2x + 5 &=& -1(x-2) \\ 2x + 5 &=& -x + 2 \\ +x & & +x \\ \hline 3x + 5 &=& 2 \\ -5 & & -5 \\ \hline 3x &=& -3 \\ \frac{3x}{3} &=& \frac{-3}{3} \\ x &=& -1 \end{array}$$

So the solutions are $x = -7$ , $x = -1$