# Number Sense Foundations

### Original by Kathy Wahl, Parkland College; edited for general use by Susan Jones

I’ve included teaching ideas in the comments; note that different individuals and class groups will have different “personalities,” so revision is a great idea -- especially if you share your experience and what worked and didn’t work.

Comments and corrections welcome -- the “live” version of this on Google Docs is <https://docs.google.com/document/d/1EO1IyUZZ22BMOpE1r-g74eiuGoV1XBznZhcUkujilfA/edit?usp=sharing>

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**1**

**Chapter 1 Adding and Subtracting Signed Numbers**

**Section 1.1 Introduction**

What is a positive number? What is a negative number? We can think about familiar things to understand these ideas. You know that if the temperature is -12° F it is very, very cold. Most thermometers give us digital temperatures now, but you have all seen an “old fashioned” thermometer:

°F

120°--

-

100°--

-

80°--

-

60°--

-

40°--

-

20°--

-

**0°--**

-

-20°--

-

-40°--

You know that -12° F is below 0° F. You know that -12° F is a colder temperature than 0° F. It follows that -12 is a smaller number than 0. Locate -12° F on this thermometer, and mark it with an arrow.

A **positive number** is a number that is \_*more than zero*  or *greater than zero*.

A **negative number** is a number that is *less than zero* or *lower than zero*

**2**

Example 1

Which is colder, 15° C or 34° C? 15 Locate both temperatures on this thermometer, and mark them with arrows.

**°C**

60°--

50°--

40°--

30°--

20°--

10°--

0°--

-10°--

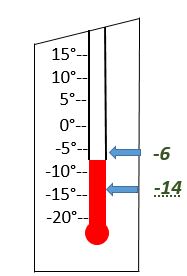
-20°--

15° C is colder than 34° C, and 15 is less than 34.

34° C is warmer than 15° C, and 34 is greater than 15.

Example 2

Which is colder, -6° F or -14° F? \_-14\_\_\_ Locate both temperatures on this thermometer, and mark them with arrows:



-14° F is colder than -6° F, and -14 is less than -6.

-6° F is warmer than -14° F, and -6 is greater than -14.

**3**

Example 3

Put these temperatures in order from smallest to largest (or coldest to warmest):

38°, 70°, -10°, 15°, -6°, 45°, 0°

-10 -6 0 15 38 45 70

Which of those temperatures are positive? 15, 38, 45, 70

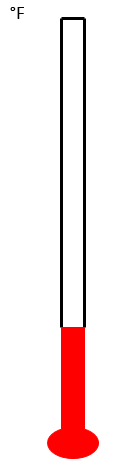
Which of those temperatures are negative? -6, -10

Is 0 a positive or negative number? Neither one.

Example 4

Make marks and label the thermometer below with appropriate numbers for a (Fahrenheit) thermometer. Make marks every 10 degrees. Try to go as low as -30° and as high as 120°.

Answers will vary – Most important concept:   that the lines and numbers are evenly spaced, no matter where the “top” is.



**4**

Now let’s think about money ☺, or the lack thereof ☹.

Example 5

In this example we are only considering the amount of money people have in their wallet. Samuel looks in his wallet and finds two tens and three ones; he has \_\_$23\_. April looks in her wallet and finds no money, but she does find a little scrap of paper that reminds her she owes her sister $12. How much money does April “have”? \_-12$\_\_

Example 6

Figure out who has more money in each of the following cases:

1. Jaden has three five dollar bills and seven one dollar bills. Jaden has \_$23\_\_\_.

Omar has a twenty dollar bill. Omar has \_\_$20\_\_\_.

Jaden has more money.

1. Kevin has only a note saying he owes his friend $40. Kevin has -40\_\_\_.

Erin has only a note saying she owes her father $50. Erin has \_-50\_\_.

\_Kevin\_ has more money.

1. Brazylle has a five dollar bill and a note reminding her she owes her mom $5. Brazylle has \_$0\_\_\_\_.

Truu has a ten dollar bill and a note reminding him that he owes his sister $15. Truu has \_\_$-5\_\_.

\_\_\_\_\_\_Brazylle\_\_\_\_ has more money.

The addition problems illustrated in (c) are: 5 + -5 =\_$0

and 10 + -15 = \_\_$-5\_\_.

**5**

Example 7

Jacob has in his wallet a twenty dollar bill, three five dollar bills, four one dollar bills, a note saying he owes his friend $10, and a note saying he owes his mom $7. How much money does Jacob “have”?

Use these pictures to help you figure it out:

$1 $1

$1 $1

$1 $1

$1 $1

$1 $1

$1 $1

$1 $1

$1 $1

**$5 $5**

**$5 $5**

**$5 $5**

**$5 $5**

**$5 $5**

**$5 $5**

**$20 $20**

**$20 $20**

**I.O.U.**

**$7**

**I.O.U..**

**$10**

The $10 would be paid w/ two $5 bills; the 7$ paid w/ the other 5 and 2 of the 1$ bills. Jacob would have $20 and two 1$ bills so he’d have $22.

Example 8

Put these numbers in order from smallest to largest:

17, -5, 3, -3, 0, 12, -14

-14, -5, -3, 0, 3, 12, 17

SECTION ONE POINT TWO:

**6**

**Section 1.2 The Number Line**

The number line is a very important tool in mathematics. We looked at thermometers in the last section, and they are very similar to number lines. Thermometers are usually vertical, but number lines are usually horizontal:

-5

-4

-3

-2

-1

0

5

1

4

3

2

When comparing two numbers on the number line, the number further to the right is greater or larger

When comparing two numbers on the number line, the number further to the left is \_\_smaller or lesser.

Example 1

Which number is greater (bigger), a or b? a\_\_

Which number is smaller, a or b? b\_ b a

**Symbols for “Greater than” and “less than”**

We have symbols for greater than (bigger than) and less than (smaller than):

Since 10 is greater than 5 we can write 10 > 5.

Since 2 is less than 7 we can write 2 < 7.

Smaller Number < Bigger Number

Bigger Number > Smaller Number

Imagine dots at the ends and the point. The side with more dots is the side with the bigger number. So:

1 < 10 one is less than ten

10 > 1 ten is greater than one

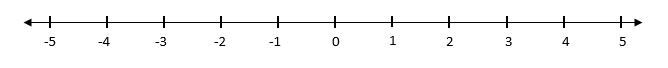
<https://www.youtube.com/watch?v=M6Efzu2slaI> – video with the famous “alligator” story for remembering which is which.

[Brainzy Games by Education.com] (2014, July 16), *Number Gators (Greater Than, Less Than Symbols Song)* [Video File] Retrieved from [https://www.youtube.com/watch?v=M6Efzu2sla](https://www.youtube.com/watch?v=M6Efzu2slaI)

**7**

Example 2

Use this number line to fill in the appropriate symbol, < or >, in each blank:



-2 < 5 0 < 3 1 > -3 -2 > -3

5 > 1 -4 < 0 -4 < -1 5 > -5

Example 3

Label this number line, counting by twos:



**0**

-2

-4

-6

-8

-10

-12

12

10

8

6

4

2

Example 4

Label this number line, counting by fives:



0

35

40

25

20

15

30

10

5

-20

-15

-10

-5

Example 5

Only two numbers are labeled on this number line. Figure out how to label the rest so that the numbers are evenly spaced:

15

-15



6

-12

18

12

9

-18

-9

-6

-3

3

0

**8**

Example 6

Fill in each blank with < or >. Refer to the number lines on the previous page as needed.

4 < 8 -6 > -15 -10 < 15 -15 < -9

-5 > -20 10 \_ > -2 -6 > -12 0 > -15

Example 7

Fill in each blank with < or >. Refer to the given number line.

b

d

0

a

c

c < a 3 > a -4 < d b > 0

-5 < 0 d > -1 a < b b > -6

Example 8

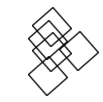
Fill in each blank with < or > if you know the following: x < y, w < 0, and w > y

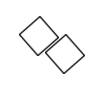
x < w 2 > w 0 > y y > x

x < 5 x < 0

**Section 1.3 Adding Signed Numbers 9**

You know how to add positive numbers. You know a variety of ways you could picture the addition

problem 2 + 5 = 7:



+

OR



+

We have to carefully pick our illustrations if we want to depict negative numbers! How can we draw negative two fingers??

We will use color in our illustrations to differentiate between positive and negative numbers. When discussing money, being “in the black” means you have a positive amount of money. Being “in the red” means you are in the hole, in other words, you “have” a negative amount of money.

In illustrations throughout this book we will use B to represent positive one, and R to represent negative one

(B for black, and R for red). You could write them in those colors if you’d like!

**10**

Example 1

Illustrate 2 + 5 = 7

**BB +   BBB BB = BBB BB BB**

Example 2

Illustrate 4 + 5 = 9

**BBBB +   BBB BB =    BBB BBB BBB**

Example 3

Illustrate -2 + -3 = -5

**RR  + RRR   = RR RRR**

Example 4

Illustrate -6 + -4 = -10

**RRRRRR   + RRRR  = RRRRR RRRRR**

**Parentheses: What do they mean(really)?**

Some textbooks will use parentheses around negative numbers to avoid having a “plus sign” and a “minus sign” right next to each other. You might see the above problem written:

-6 + (-4) = -10 or even (-6) + (-4) = -10

Parentheses are often used to “wrap” a number or an expression so that it’s easier to figure out what to do.

They are all ways of writing the same problem. Having parentheses involved does **NOT** automatically make it a multiplication problem.

**-6 + (-4) indicates addition**

When is it a multiplication problem? When the two numbers are right next to each other.

-6(-4) indicates multiplication because the 6 is right next to the parenthesis, with no other operation

between them.

Multiplication is so common that In math, if things are just next to each other, it’s as if there were an invisible multiplication sign there. In algebra, a multiplied by b can be written as just *ab*

**If there’s another operation… that’s what you do. So (-6) + (-4) = -10** **is an addition problem because it’s got an addition sign there.**

**11**

Example 5

Draw an illustration for each of these problems using R’s and B’s. Don’t forget to state the answer to each problem too! (If you’d like, use red for negative and black for positive.)

1. -1 + -7 = -8 R + RR RR RR R = RR RR RR RR
2. 3 + 8 BBB + BB BB BB BB = BB BB BB BB BB B
3. -6 + (-2) RRR RRR + RR = RRR RRR RR
4. -4 + 0 RRRR + = RRRR
5. (-3) + (-4) RRR + RRRR = RRRRRRR
6. 4 + 7 BB BB + BB BB BB B = BB BB BB BB BB B

Do you see the pattern in all of these problems? You have long known that when you add two positive numbers you get a bigger positive number. Now you see that when you add two negative numbers you get a “bigger” negative number. (Why did I put “bigger” in quotes?)

This is what happens when you add any like things together:

* When you add 3 cats to 2 cats you get 5 cats!
* When you sell 2 hot dogs then 3 more hot dogs you have sold 5 hot dogs.
* When you put together 2 R’s and 3 R’s you get 5 R’s,

Or in mathematical terms: -2 + -3 = -5

***Or, “It’s same to same in the addition game.”***

**But don’t two negatives make a positive????**

     Not when you’re adding.   **Two wrongs don’t make a right, do they?**

When you’re not sure, it helps to imagine the number line or a football field.    
**Loss plus loss is … more loss!!!**

(And another pattern:   the rules & procedures for adding and subtracting are pretty much always \*really different\* than the rules for multiplying and dividing.   Two different kinds of things. )

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Let’s think of some real life illustrations that support the fact that -2 + -3 = -5

1. If the temperature **drops** 2° I can think of that change in temperature as -2°.

If the temperature drops 2° one hour, then 3° the next hour, it has dropped a total of 5°:

-2 + -3 = -5

Make up your own! Money, golf, anything that has a “zero” place that you can go below will do. Yes, you can look back at the examples for ideas.

1. Answers will vary. Remind students that they can use temperature or money – or yards lost in American football or pounds lost on a diet or “under par” in golf (or something else if they can think of it… and they change the starting point).   
    Some students may have to go back to the example and change some numbers; others will be able to use more imagination.

3.

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Example 6

Do these addition problems. You do not have to illustrate them.

1. (-7) + (-2) -9
2. 6 + 3 9
3. -5 + 0 -5
4. 4 + 6 10
5. -9 + (-1) -10
6. 20 + 40 60
7. -20 + -10 -30
8. -4 + -3 + -6 -13

Example 7

Fill in each blank with <, >, or = to make a true statement:

1. -2 + -5 = -4 + -3 d) -3 + 0 + -6 < -4 + -4
2. 4 + 0 < 3 + 2 e) 6 + 1 > -6 + -1
3. -4 + -2 > -3 + -4 f) (-1) + (-2) + (-3) + (-4) = -10

**14**

**Section 1.4 Addition on the Number Line**

We differentiated between positive and negative numbers by using black for positive numbers, and red for negative numbers. We have also looked at signed numbers on the number line, on which positive numbers are to the right of zero, and negative numbers are to the left of zero.

How can we use the number line to illustrate addition?

Example 1

Let’s look at 2 + 3 = 5 on the number line:

| | | | | | | | | | |

-4 -3 -2 -1 0 1 2 3 4 5 6

We begin at 2, then add 3 by moving 3 spaces to the **right**. Notice that since 2 is a positive number, it is to the

Right of zero; then we move further to the \_Right when we add the positive number 3.

Example 2

Illustrate 4 + 3 = 7 on the number line:

| | | | | | | | | | |

-2 -1 0 1 2 3 4 5 6 7 8

Example 3

Illustrate -2 + -4 = -6 on the number line:

| | | | | | | | | | |

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2

We begin at -2, then add -4 by moving 4 spaces to the **left**. Notice that since -2 is a negative number, it is to

the \_left\_ of ze**r**o; then we move further to the left\_ when we add -4 (**negative** 4).

**15**

The **Commutative Property of Addition** states: **a + b = b + a**

Example 4

Illustrate 2 + 9 = 11 on one number line, then 9 + 2 = 11 on the other number line:

Start at blue

| | | | | | | | | | | | | |

-1 0 1 2 3 4 5 6 7 8 9 10 11 12

Start at blue

| | | | | | | | | | | | | |

-1 0 1 2 3 4 5 6 7 8 9 10 11 12

Example 5

Illustrate (-3) + (-5) = -8 on one number line, then (-5) + (-3) = -8 on the other number line:

Start at blue

| | | | | | | | | | | | | |

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5

Start at blue

| | | | | | | | | | | | | |

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5

Example 6

How would you illustrate 5 + 0 = 5 on the number line? Illustrate 5 + 0 = 5 on one number line, then

0 + 5 = 5 on the other number line:

| | | | | | | | | | | | | |

-1 0 1 2 3 4 5 6 7 8 9 10 11 12

Start at blue

| | | | | | | | | | | | | |

-1 0 1 2 3 4 5 6 7 8 9 10 11 12

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Example 7

Compute each of the following sums using a number line sketch AND an illustration with R’s or B’s.

(Recall that R = -1 and B = 1.) Be sure to state the answer!

Start at blue

1. -4 + (-2) =

| | | | | | | | | | | | | |

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4

RRRR + RR = RRRRRR -6

1. 1 + 7 =

| | | | | | | | | | | | | |

-1 0 1 2 3 4 5 6 7 8 9 10 11 12

Start at blue

B + BBB BBB B = BBB BBBB 8

1. (-6) + (-4) =

| | | | | | | | | | | | | |

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2

RRRR RR + RRRR = RR RRR RR RRR

-10

Example 8

Fill in each blank with <, >, or = to make a true statement:

1. -4 + -7 = -6 + -5 d) 4 + 5 > -5 + (-4)
2. (-2) + (-5) < -3 + -1 e) -8 + 0 < -2 + -4
3. -4 + 0 < 2 + 5 f) -3 + -8 + -5 < \_ -7 + -1 + -4

**17**

Number line illustrations are very helpful when adding signed numbers, but they would not be practical if we were adding very large numbers. We can summarize what we have seen in these illustrations with this rule:

**RULE**

**TO ADD TWO NUMBERS THAT HAVE THE SAME SIGN, ADD. THE ANSWER SHOULD HAVE THE SAME SIGN AS THE TWO NUMBERS.**

Example 9

Think about the above rule as you compute these sums:

1. -14 + (-3) = -17
2. -33 + -44 = -77
3. 42 + 34 = 76
4. (-156) + (-347) = -503

**18**

**Section 1.5 Adding Positive and Negative Numbers**

If the two numbers are the same sign, we’ll add them – and keep that sign. The “number part” (absolute value) is bigger, but of course a “bigger” negative is an even smaller number because it’s below zero.

**But what if we’re adding positive to negative??? They’re opposites!!!**

Before we can proceed, we need to consider the idea of **opposites**. For example,

**-3 is the opposite of 3.**

This makes sense because -3 is three units \_left\_ of zero, while 3 is three units \_right\_ of zero.

And … left is the opposite of right, right?

**5 is the opposite of -5 and**

**-5 is the opposite of 5**

This makes sense because 5 is five units right of zero, while -5 is five units left of zero.

Example 1

Compute 1 + -1.

We begin at 1 on the number line. To add -1 we jump one unit to the left, landing at 0:

| | | | | 1 + -1 = 0

-2 -1 0 1 2

This makes intuitive sense. Let’s say you have $1, then you lose $1. You would have $0 !!

*Two of the same “number part”(absolute value) with opposite signs will add up to zero.*

Example 2

Compute -3 + 3 on the number line:

| | | | | | | | | | |

-5 -4 -3 -2 -1 0 1 2 3 4 5

Start at blue

So, -3 + 3 = \_0\_\_\_

**19**

Example 3

Compute -6 + 2 on the number line:

Start at blue

| | | | | | | | | | | | |

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5

-6 + 2 = \_-2\_\_\_

Example 4

How would we illustrate -6 + 2 using R’s and B’s?

-6 + 2 =

RRR RRR + BB =

RRR R~~RR~~ + ~~BB~~ = RRR R

Example 5

Compute 8 + (-3) using a number line illustration, then check using R’s and B’s.

Start at blue

| | | | | | | | | | | | | | | | | |

-1 0 1 2 3 4 5 6 7 8 9 10 11 12

BB BBB BBB + RRR

BB BBB ~~BBB~~ + ~~RRR~~

8 + (-3) = 5

**20**

Example 6

Compute each of the following by making a number line sketch:

1. 4 + (-5) = | | | | | | | | | |

-4 -3 -2 -1 0 1 2 3 4 5

1. 6 + -1 = | | | | | | | | | |

-2 -1 0 1 2 3 4 5 6 7

Start at blue

1. -7 + 5 = | | | | | | | | | |

-7 -6 -5 -4 -3 -2 -1 0 1 2

1. -20 + 50 = | | | | | | | | | |

-30 -20 -10 0 10 20 30 40 50 60

Example 7

Compute each of the following by illustrating with R’s and B’s:

(students may cross out pairs of negative R’s and positive B’s)

1. 5 + -5 = BBBBB + RRR RR = 0

~~BBBBB~~ + ~~RRRRR~~

1. -3 + 1 = RRR + B = RR = -2

RR~~R~~ + ~~B~~

1. 7 + (-5) = BBBBB BB + RRR RR = BB

~~BBBBB~~ BB + ~~RRR RR~~ = BB = 2

1. -4 + 8 = RR RR + BBBB BBBB = BBBB = 4  
   ~~RR RR~~ + ~~BBBB~~ BBBB

**21**

What have you noticed about adding a positive number and a negative number?

For one thing, when adding numbers that have different signs, movement on the number line is in two directions. You might start right of zero with a positive number, then move left to add a negative number.

When using the R and B illustration, we see that one R and one B cancel each other since -1 + 1 = 0.

Our illustrations are great, but they would not be practical if we were adding large numbers.

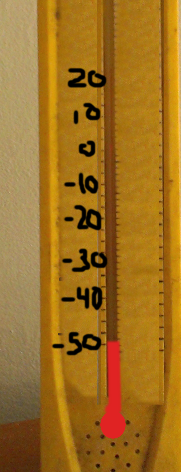
Here is a rule that tells us how to add numbers that have different signs.

**RULE**

**TO ADD TWO NUMBERS THAT HAVE DIFFERENT SIGNS, SUBTRACT. THE ANSWER SHOULD HAVE THE SAME SIGN AS THE “BIGGER” NUMBER.**

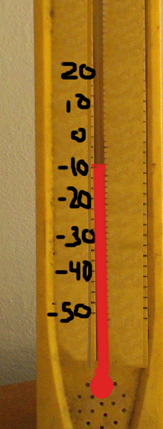
**?????   What, exactly,  do I subtract?**

For this part, we only care about the “number part,” not the sign.   It’s called the “absolute value.”

 What’s -50 + 40?

On my trip to Alaska, the temperature started at 50 below zero… but it got 40 degrees warmer! We added warmth.

50 -- the ‘number part’ -- is bigger than 40; it’s going to start our subtraction problem.   The big absolute value (number part)  goes on top, the smaller underneath; subtract.

 50

        − 40

10

Then it’s time to think.   Did we gain more, or lose more?

-50 + 40

The negative “number part” was bigger, so… our final answer is a negative 10…   - 10.   We didn’t get all the way back up to zero.

Video explaining adding numbers with different signs (8 minutes):

<http://www.resourceroom.net/math/DiffSignAdd/DiffSignAdd.html>

Curious? Video explaining absolute value (what we’re calling “number part”) (5 minutes):

<http://www.resourceroom.net/math/AbsoluteValueBasics/AbsoluteValueBasics.html>

**22**

Example 8

Use the above rule to compute -40 + 25.

Since we are finding the sum (adding) numbers that have different signs, we actually SUBTRACT the numbers! 40

− 25

15

We decide that the answer should be **negative**, since 40 is **bigger** than 25, and the sign “on 40” is **negative**.

So, -40 + 25 = -15

Example 9

Use the rule to compute each of these:

1. -10 + 7 -3 d) 8 + -5 3
2. 14 + -3 11 e) -6 + 1 -5
3. -33 + 11 \_-22 f) 45 + (-35) 10

Example 10:   Make up 3 problems of your own that add a positive and a negative number, and have a positive answer.

(answers will vary ☺)

Example 11: Make up 3 problems of your own that add a positive and a negative number, and have a negative answer.

Example 12:   Make up a story adding a positive and negative amount.   It can be about temperatures, money, or anything that can be below zero.

**23**

**Section 1.6 Mixing Up Types of Addition**

When you look at an addition problem with signed numbers, the first thing you must consider is whether you are adding numbers that have the **same sign**, such as

(-2) + (-4) = -6

or numbers that have **different signs**, such as

-5 + 8 = 3

**ADDING NUMBERS THAT HAVE THE SAME SIGN:**

When adding numbers that have the **same sign**, all movement on the number line is in the **same direction**:

(-2) + (-4) = -6 | | | | | | | | | | | |

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1

We begin 2 units **left** of zero, then jump 4 more units to the **left**, landing at -6 which is 6 units **left** of zero!

**ADDING NUMBERS THAT HAVE DIFFERENT SIGNS:**

When adding numbers that have **different signs**, the movement on the number line is in **two directions**:

-5 + 8 = 3 | | | | | | | | | | | |

-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4

We begin 5 units **left** of zero, then jump 8 units to the **right**, landing at 3.

We moved three **more units to the right** **than we did to the left**, and that is why we ended up at 3, which is a positive number. We have to subtract to find out how many more units we went right than left.

**24**

While the number line is a wonderful tool, it is not always practical to illustrate every addition problem. As we mentioned in the last two sections, it is particularly hard to illustrate problems with very large numbers. For that reason we summarized both types of addition (same sign, different signs) with rules. Here are the rules again:

**RULE**

TO ADD TWO NUMBERS THAT HAVE THE **SAME SIGN,** **ADD**. THE ANSWER SHOULD HAVE THE SAME SIGN AS THE TWO NUMBERS.

**RULE**

TO ADD TWO NUMBERS THAT HAVE **DIFFERENT SIGNS**, **SUBTRACT**. THE ANSWER SHOULD HAVE THE SAME SIGN AS THE “BIGGER” NUMBER.

Example 1

Compute 7 + (-10)

We are adding numbers that have **different signs**, so we **subtract**.

We subtract: 10 - 7 = 3. We know that the answer should be negative because 10 is bigger than 7 and the sign on 10 is negative.

So, 7 + (-10) = -3

Example 2

Compute -20 + -11

We are adding numbers that have the **same sign**, so we **add**.

We add: 20 + 11 = 31. We know that the answer should be negative because the numbers we are adding are both negative.

So, -20 + -11 = -31

**25**

Before we do any more examples, let’s summarize our two rules with this song, to the tune of “Row, Row, Row Your Boat”:

Same sign add and keep, different signs subtract; keep the sign of the bigger

number then you’ll be exact!

Example 3

Do each of these problems using the rules. Check each answer with a number line sketch.

Start at blue

1. 3 + 9

| | | | | | | | | | | | | |

-1 0 1 2 3 4 5 6 7 8 9 10 11 12

Same sign, add & keep so 9 + 3 = 12

1. 6 + -3

| | | | | | | | | | | | | |

-1 0 1 2 3 4 5 6 7 8 9 10 11 12

Different sign, subtract: “bigger” number is 6 so 6 + -3 = 3

1. (-5) + (-4)

| | | | | | | | | | | |

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1

Same sign, add & keep so -5 + -4 = -9

1. -10 + 2

| | | | | | | | | | | |

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1

Different sign, subtract: 10 -2 = 8, but “bigger” number is the -10, so answer is -8

**26**

Example 4

Do each of these problems using the rules. Check each answer with an illustration using R’s and/or B’s:

1. -5 + 1 Different Sign, subtract; keep negative because “5” is bigger than “1” -5 + 1 = -4

RRRR~~R~~ + ~~B~~

1. -3 + (-2) Same Sign, add & keep the sign so -3 + (-2) = -5  
   Remember that parentheses “wrap” the number but don’t mean that you have to do anything to it.RRR + RR
2. 4 + -4 Different sign, subtract; answer is 0 (neither one is bigger; 0 is neither positive or negative.)

~~BBBB~~ + ~~RRRR~~

Example 6

Compute each of the following:

1. 14 + 10 Same Sign, add & keep:  
   14 + 10 = 24
2. 7 + (-18) Different sign, subtract; bigger “number part” is 18 so answer is negative.

18 – 7 = 11 7 + (-18) = -11

1. -14 + (-17) Same Sign, add & keep

-14 + -17 = -31

1. -30 + 50 Different Sign, subtract; 50 is bigger “number part” so answer is positive. 50 – 30 = 20 -30 + 50 = 20

**27**

When adding a string of numbers, if it is ALL addition, we can add the numbers in any order we choose.

This is because of the Commutative Property of Addition: a + b = b + a.

When the string of numbers we are adding includes both positive and negative numbers, it might be easiest to first add all of the positive numbers together, and all the negative numbers together, then add those results:

Example 7 5 + (-9) + (-2) + 7

= 12 + (-11)

= 1

Example 8

Add each of these strings of numbers:

1. -6 + -9 + 4 + -2 + 5

9 + -17 = -8

1. 14 + (-7) + 11 + (-6) + (-3)

25 + (-16) = 9

**Section 1.7 Subtracting Signed Numbers 28**

Subtraction is the opposite of addition.

<http://library.parkland.edu/friendly.php?s=mat094cas>    has several Powerpoints that you might find useful for learning and reviewing this material.

How do we compute 3 − 7? Let’s look at it on the number line:

| | | | | | | | | |

=5 -3 -2 -1 0 1 2 4

3

-4

Why did we move left seven spaces? Three minus seven means a decrease of seven, hence seven spaces to the left (beginning at three).

So, 3 − 7 = -4

Compare that to the problem 3 + -7 = -4.

Are they the same problem? \_\_Yes ☺ \_\_\_\_

In fact, we can rewrite every subtraction problem as an addition problem. That way we don’t have to learn a whole new set of rules for subtraction!

Change a − b (“a minus b”)

to a + -b (“a plus the opposite of b”)

This takes two changes. Change subtraction (minus) to addition (plus) AND change

b to the opposite of b.

Example 1 5 − 11

becomes 5 + -11 which equals -6

Example 2 8 − 2

becomes 8 + -2 which equals 6

Example 3 -2 − 9

becomes -2 + -9 which equals -11

Note: You do not have to rewrite the problem to change it. Just change the minus sign into a plus sign then change the sign on the second number.

**29**

Change each of these subtraction problems to an addition problem (and change the sign of the second number) then compute the answer:

1. 2 – 9 2 – 9
2. -3 – 1 -3 – 1
3. 6 – 5 6 – 5
4. -6 – 5 -6 – 5
5. 3 – 8 3 − 8
6. -10 – 3 -10 − 3

How does this apply to a problem like 4 − -6?

We still change the subtraction to addition and change the sign of the second number:

Change a − - b (“a minus negative b”)

to a + + b (“a plus positive b”)

This is still making two changes. You can just write a + b

Example 4 2 − - 8

becomes 2 + + 8 which equals 10

Example 5 -4 − -6

becomes -4 + + 6 which equals 2

Example 6 -1 − (-4)

becomes -1 + (+4) which equals 3

Remember, you do not have to rewrite the problem to change it. Just change the minus sign into a plus sign and change the sign of the second number.

**30**

In summary:

Every time we encounter subtraction we change the **minus** sign into a **plus** sign **and** change the sign of the second number.

Change each of these subtraction problems into addition problems (and change the sign of the second number) then compute the answer:

1. 4 − - 6 4 ++6 Same Sign: 4 + 6 = 10
2. -2 − (-5) -2 ++ 5 Different sign: -2 +5 = 3
3. -6 – 1 -6 +-1 Same Sign: -6 + -1 = -7
4. 2 − (-3) 2 ++3 Same sign: 2 + 3 = 5
5. 8 – 10 8 +-10 Different sign: -2
6. -3 – 11 -3 + -11 Same Sign… -14
7. -8 − (-4) -8 ++ 4 Different Sign: -4
8. 5 – 1 5 + -1 Different sign: -4
9. 6 − - 2 6 ++ 2 Same Sign: 8
10. -7 – 4 -7 + -4 Same Sign: -11

**31**

This next set of problems is a mix of addition and subtraction problems.

CHANGE THE SUBTRACTION PROBLEMS.

DO NOT CHANGE THE ADDITION PROBLEMS.

1. -7 + 10 ***DON’T CHANGE!   Different sign…            3***
2. -3 – 8 ***Change Change! -3 + -8 Same Sign… -11***
3. 3 − - 11 Change Change! 3 ++ 8 Same Sign… 11
4. -3 + (-7) Don’t Change! Same sign… -10
5. -6 − (-2) Change Change! -6 ++2 Different Sign… -4
6. 4 + -9 Don’t Change! Different Sign… -5
7. 8 – 2 Change Change! 8 + -2 Different Sign … 6
8. -20 + 15 Don’t Change! Different sign… -5
9. -3 − -12 Change Change! -3 ++ 12 Different Sign… 9
10. -4 + -6 Don’t change! Same Sign… -10

**32**

In this last set of problems you will encounter addition and subtraction within the same problem. **First** go through the problem and change each **subtraction** to addition (and change the sign of the second number). **Then** begin computing the answer.

1. -5 − 3 + 6 − (-3) -5 + -3 + 6 ++3

-8 + 9 Different Sign… 1

1. 10 + (-20) + 30 – 40 10 + -20 + 30 + -40

40 + -60 Different Sign… -60

1. 10 − 14 + -5 10 + -14 + -5

10 + -19 = -9

1. -6 + -5 − 4 + -3 − 2 + 1 -6 + -5 + -4 + -3 + -2 + 1

-20 + 1 Different Sign… -19

**33**

**Chapter 1 Review**

We have seen that the number line helps us understand many things about numbers, especially signed numbers. We learned that when comparing two numbers, the larger number is the one further to the right on the number line, and the smaller number is the one further to the left on the number line.

Example 1

Which number is larger, -5 or -9?

The number line shows us that -5 is larger than -9 because it is further right than -9.

| | | | | | | | | | | | | | | | | | | | |

-9 -5 0

We can write -5 > -9 or -9 < -5

Example 2

Write < or > in each blank to make a true statement; use this number line:

 a < c

e > b

d > a

e < c

Example 3

Write < or > in each blank to make a true statement.

0 > -2

-5 > -3

5 > -7

2 < 11

-6 < 6

-2 > -8

**34**

We learned two ways to illustrate adding signed numbers. We used black to indicate positive numbers,

and specifically, B = 1. We used red to indicate negative numbers, and specifically, R = -1.

-3 + 5 = 2: ~~RRR~~ + ~~BBB~~BB = 2

We also learned how to illustrate addition problems on the number line.

-3 + 5 = 2: | | | | | | | | | | |

-5 -4 -3 -2 -1 0 1 2 3 4 5

Example 4

Compute each of the following; illustrate each problem with R’s and/or B’s:

1. 4 + 7

BBBB + BBBBBBB = BBBBBBB = 11

1. -2 + 6

~~RR~~ + ~~BB~~BBBB = 4

1. (-3) + (-5) RRR + RRRRR = RRRRRRRR = -8

Example 5

Compute each of the following; illustrate each problem on a number line:

Start at blue

1. (-7) + (-3)

| | | | | | | | | | | |

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1

1. 2 + (-6)

| | | | | | | | | | | |

-4 -3 -2 -1 0 1 2 3 4 5 6 7

1. -4 + 5

| | | | | | | | | | | |

-4 -3 -2 -1 0 1 2 3 4 5 6 7

**35**

After doing many addition problems with illustrations, we realized that we could state rules that would allow us to do addition problems without a number line or circle illustration.

**RULE**

TO ADD TWO NUMBERS THAT HAVE THE **SAME SIGN, ADD** . THE ANSWER SHOULD HAVE THE SAME SIGN AS THE TWO NUMBERS.

**RULE**

TO ADD TWO NUMBERS THAT HAVE **DIFFERENT SIGNS, SUBTRACT**. THE ANSWER SHOULD HAVE THE SAME SIGN AS THE “BIGGER” NUMBER.

Example 6

Compute each of the following using the rules. Remember, the first thing you need to look at is whether you are adding numbers that have the **same sign**, or numbers that have **different signs**, so you know which rule to use!

1. 42 + -16 26
2. -18 + -22 -40
3. 32 + 47 79
4. -80 + 30 -50
5. (-28) + (-11) -39

Example 7

Fill in each blank with <, >, or =, to make a true statement:

-4 + 5 \_ > -5 + 4 -3 + -6 = (-2) + (-7)

(-11) + 2 < 3 + (-7) -2 + 0 > 5 + -8

**36**

Finally, we tackled subtracting signed numbers. We did that by changing every subtraction problem

into an equivalent addition problem:

CHANGE THE MINUS SIGN INTO A PLUS SIGN **AND** CHANGE THE SIGN OF THE SECOND NUMBER.

The subtraction problem a − b

becomes a + -b

The subtraction problem a − -b

Becomes a + +b (or just a + b)

Example 8

Change each of these subtraction problems into the equivalent addition problem and compute:

(Change the minus to plus AND change the sign of the second number.)

1. 7 − 9 7 + -9 = -2 c) 4 − (-5) 4 ++ 5 = 9
2. -2 − (-5) -2 ++5 = 3 d) -3 – 7 -3 + -7 = -10

Example 9

Compute each of the following. Make sure you **do not** change addition problems!

1. -7 + (-10) -17 c) -30 – 40 -30 + -40 = -70
2. 13 − (-13) 13 ++13 = 26 d) -2 + 8 6

Example 10

Compute the following. First change the subtraction problems to equivalent addition problems.

1. 3 − 8 − (-4) + -7 + 4 − 9 b) 1 − 2 − 3 − 4 + -5 + 6 + 7

3 + -8 + + 4 + -7 +4 + -9 1 + -2 + -3 + -4 + -5 +6 +7

-24n + 11 = -13 -14 + 14 = 0